Two-Phase Flow in Fractured Rocks: Modeling and Numerical Difficulties

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in collaboration with

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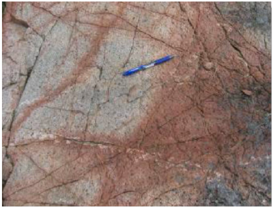
InS^{3}PiRE

Porquerolles, May 11, 2022

Motivation

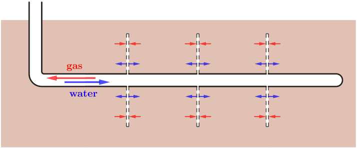
Principle characteristics:

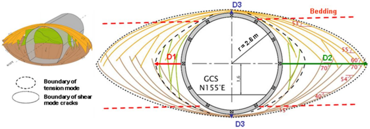
- Exist at many scales: from few *cm* to *km* (faults)
- Very large $\frac{\text{length}}{\text{width}}$ ratio $\Rightarrow$ DFN, DFM models
- Extreme contrast in hydrodynamical properties:
  permeability, capillary pressure $\Rightarrow$ strong nonlinear coupling

Industrial applications:

- Tight gas and oil extraction
- Underground nuclear waste storage
- ...

Nonlinearity + heterogeneity





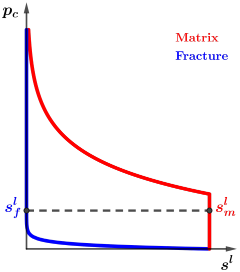
Discontinuous p_c curves $\Rightarrow$ saturation jumps:

- capillary-driven spontaneous imbibition
- capillary barrier

Strong nonlinear coupling at *mf* interface

Focus of this talk:

- Design of the two-phase DFM models
- Improved nonlinear solvers



Outline

Single-phase DFM models

- Drains and barriers

Two-phase DFM models

- Drains become barriers?!

Nonlinear solver

- Improved primary variable selection

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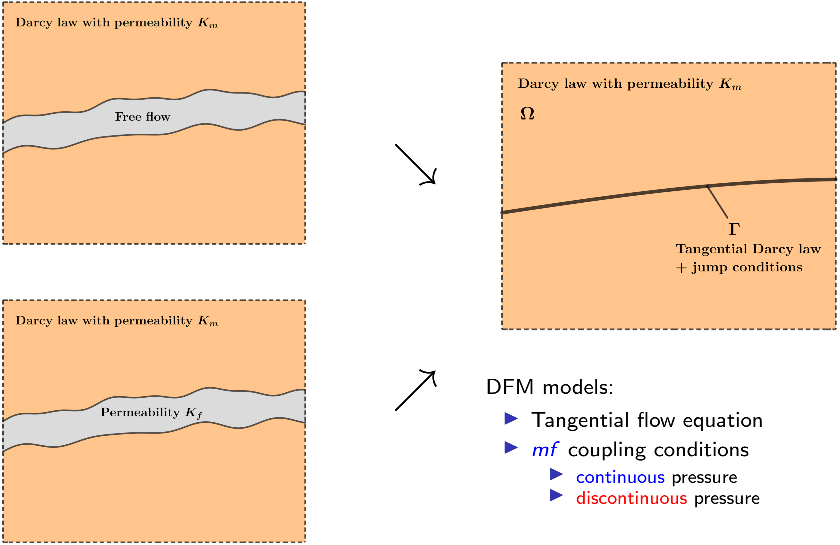
Dimension reduction



DFM models:

- Tangential flow equation
- *mf* coupling conditions
  - continuous pressure
  - discontinuous pressure

Single-phase DFM models

Coupled flow in matrix and fracture network

$$\begin{cases} \mathrm{div}\mathbf{q}_m = 0 \\ \mathrm{div}_\tau \mathbf{q}_f = \mathbf{q}_m|_{\Gamma^+} \cdot \mathbf{n}^+ + \mathbf{q}_m|_{\Gamma^-} \cdot \mathbf{n}^- \end{cases}$$

Darcy law

$$\mathbf{q}_m = -\frac{K_m}{\mu}(\nabla p_m - \rho \mathbf{g})$$
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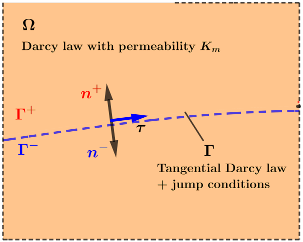

Continuous pressure model[1]

- No pressure jump across Γ

$$p_m|_{\Gamma^+} = p_m|_{\Gamma^-} = p_f.$$

with $p_m \in H^1(\Omega) \cap H^1(\Gamma)$

Discontinuous pressure model[2,3]

- Pressure jump/flux relation on $\Gamma^\pm$

$$\mathbf{q}_m|_{\Gamma^\pm} \cdot \mathbf{n}^\pm = -\frac{K_{f,n}}{\mu}\left(\frac{p_m|_{\Gamma^\pm} - p_f}{d_f/2}\right)$$

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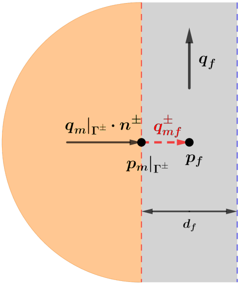

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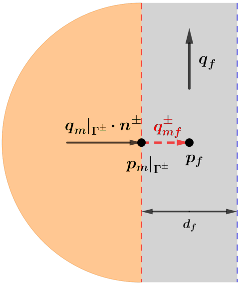

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- Continuum of models

$$\mathbf{q}_m|_{\Gamma^\pm} \cdot \mathbf{n}^\pm = -\frac{K_{f,n}}{\mu}\left(\xi \frac{p_m|_{\Gamma^\pm} - p_f}{d_f/2} + (1-\xi)\frac{p_m|_{\Gamma^\mp} - p_f}{d_f/2}\right)$$

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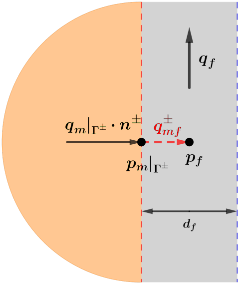

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Cont. pressure is $\frac{K_{f,n}}{d_f} \to +\infty$ limit of disc. pressure model

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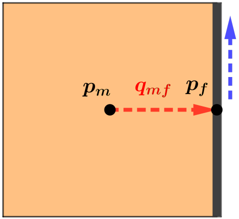
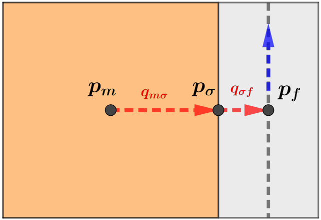
Discretized models

Example of TPFA FV discretization

Cont. pressure



Disc. pressure



Dofs: cells, frac.-faces, frac. intersections

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+ *mf*-interfaces

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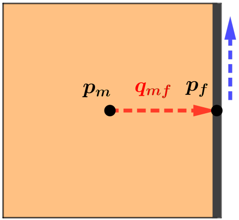

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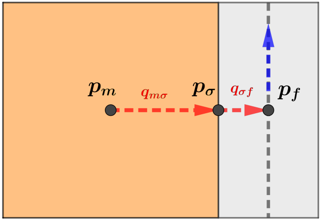

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Local elimination of *mf*-interfaces using

$$q_{m\sigma} + q_{f\sigma} = 0.$$

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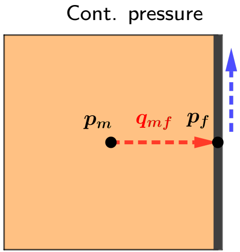

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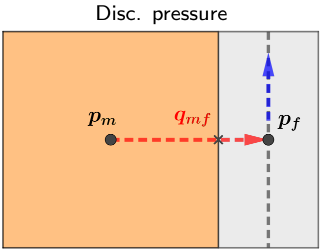

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$$q_{mf} = \frac{T_{m\sigma} T_{f\sigma}}{T_{m\sigma} + T_{f\sigma}} (p_m - p_f)$$

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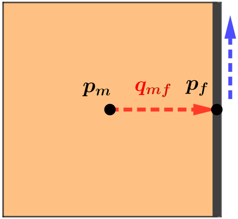

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Same number of dof! But,

- no local elimination for nodal schemes;
- not trivial for two-phase flow.

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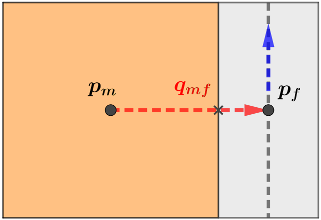

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[1] Kumar, List, Pop, Radu, 2017

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Two-phase DFM model, $\alpha = l, g$,

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Relative permeability upwinding

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Alternative: formal upscaling[1]

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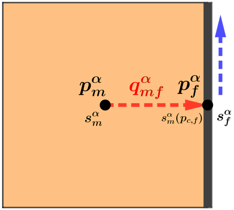

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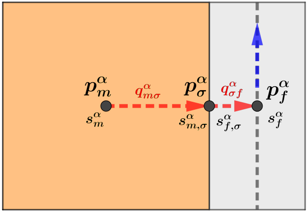

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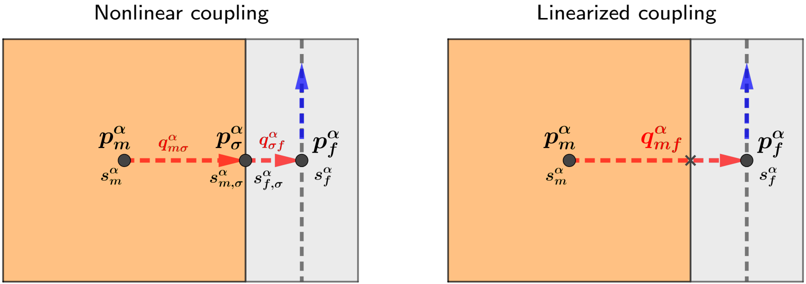
Disc. pressure model with the linearized coupling



Linearized coupling:

$$q^{\alpha}_{mf} = k^{\alpha}_{r,m}(s^{\alpha}_{m})\,(V^{\alpha}_{mf})^{+} + k^{\alpha}_{r,f}(s^{\alpha}_{f})\,(V^{\alpha}_{mf})^{-}$$

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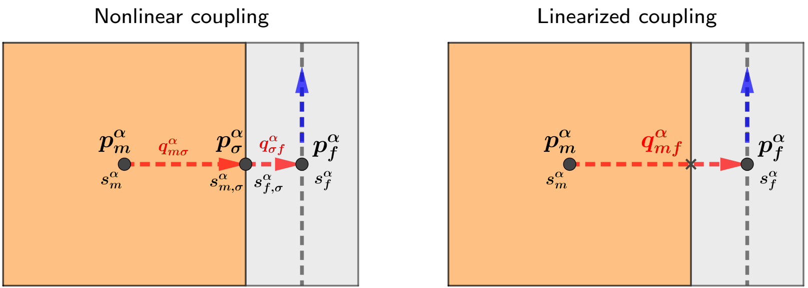

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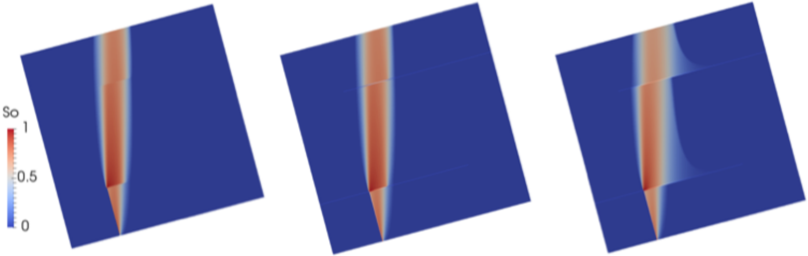
Remark:

- p_σ^α is eliminated assuming continuity of the *full Darcy flux*.
- This is equivalent to assume that $k_r(s(p_c))$ is continuous.

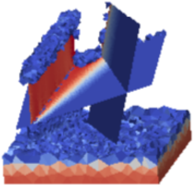
Experimenting with the two-phase DFM models

Numerical comparison and validation of two-phase DFM models[1,2]



Cases:

- In 2D and 3D;
- Fractures acting like drains and barriers, including capillary ones
- Validation using equi-dimensional model



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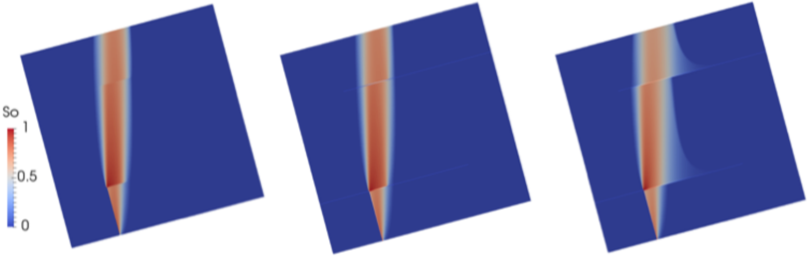

Conclusions:

- Nonlinear disc. pressure model approximate well the equi-dimensional one
- Nonlinear transmission condition is computationally challenging
- In some situation only the nonlinear disc. pressure model provides an acceptable solution.

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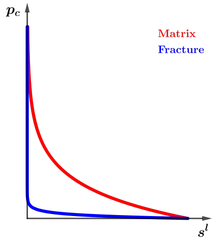
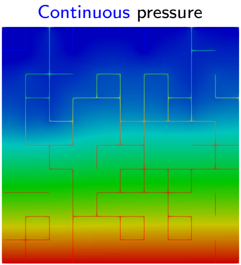
Validity of cont. pressure models: drains becomes barriers?

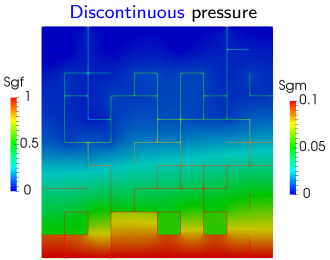
Test case: Drying of a damaged zone

- Domain $\Omega = (0, 10m)^2$
- Fracture width $d = 1mm$
- Permeability contrast $K_f / K_m = 10^4$
- Capillary pressure contrast
- Boundary conditions

$$\begin{array}{lll} \text{Saturated top:} & s_m^l = 1, & p^g = 1atm \\ \text{Dry bottom:} & s_m^l = 0.9, & p^g = 1atm \end{array}$$

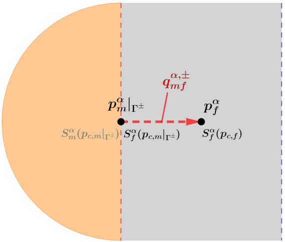


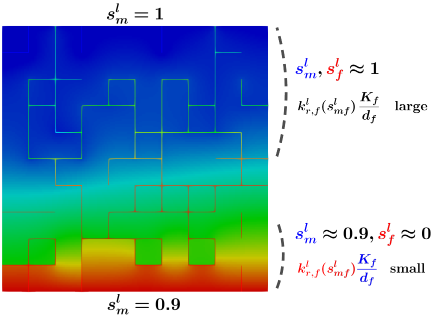




- Disc. pressure model: some fractures acts as barriers. Why?

Fracture as a capillary barrier





Pressure jump/flux relation for liquid

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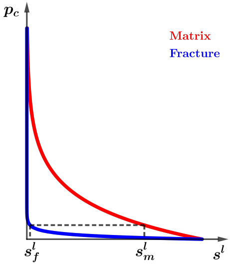
$$s_{mf}^\alpha = \begin{cases} S_f^\alpha(p_{c,m}|_{\Gamma^\pm}), & p_m^\alpha|_{\Gamma^\pm} - p_f^\alpha \geq 0, \\ S_f^\alpha(p_{c,f}), & p_m^\alpha|_{\Gamma^\pm} - p_f^\alpha < 0. \end{cases}$$



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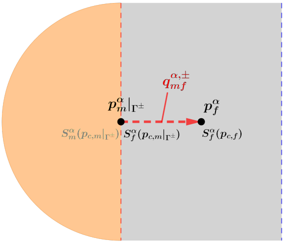
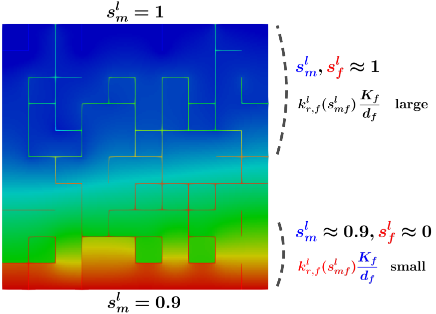




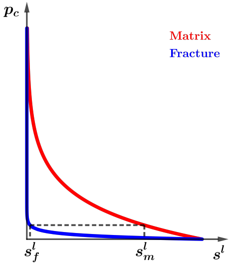
Pressure jump/flux relation for liquid

$$\mathbf{q}_m^l|_{\Gamma^\pm} \cdot \mathbf{n}^\pm = -k_{r,f}^l(s_{mf}^l)\frac{K_f}{\mu^l d_f/2}\left(p_m^l|_{\Gamma^\pm} - p_f^l\right)$$

$$s_{mf}^\alpha = \begin{cases} S_f^\alpha(p_{c,m}|_{\Gamma^\pm}), & p_m^\alpha|_{\Gamma^\pm} - p_f^\alpha \geq 0, \\ S_f^\alpha(p_{c,f}), & p_m^\alpha|_{\Gamma^\pm} - p_f^\alpha < 0. \end{cases}$$

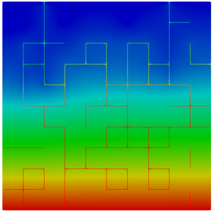
$d_f \to 0$ asymptotic model

- Cont. pressure $\to$ homogeneous matrix
- Disc. pressure $\to$ something else



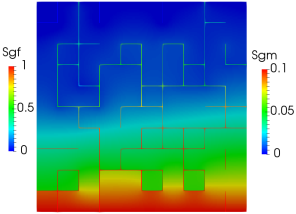
Linearized disc. pressure model

Cont. pressure



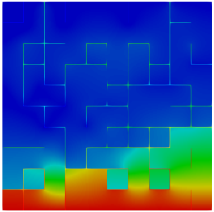
No capillary barriers

Disc. pressure



Reference

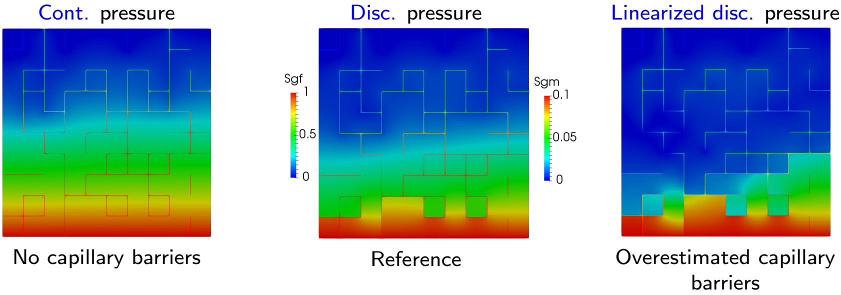
Linearized disc. pressure



Overestimated capillary barriers

[1] Aghili, de Dreuzy, Masson, Trenty, 2021

Linearized disc. pressure model



Observations:

- Highly permeable fractures may act as barriers. Model with the nonlinear coupling is required.
- Compositional models with Fickian diffusion reduce the barrier effect[1]

[1] Aghili, de Dreuzy, Masson, Trenty, 2021

Outline

Single-phase DFM models

- Drains and barriers

Two-phase DFM models

- Drains become barriers?!

Nonlinear solver

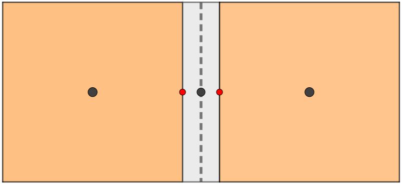
- Improved primary variable selection

Improving nonlinear convergence

Nonlinear *mf* coupling challenges the robustness of Newton's method.

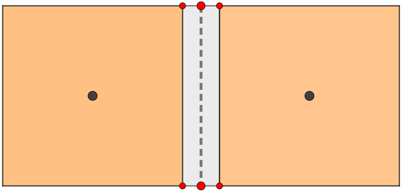
Possible solutions?

Discretization using face dofs



Local nonlinear elimination[1]

Discretization using nodal dofs



Improved primary variable selection[2]

[1] Aghili, B., Hennicker, Masson, Trenty, 2019
[2] B., Groza, Jeannin, Masson, Pellerin, 2017

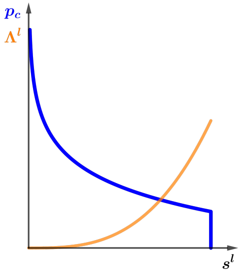
PV selection in the homogeneous two-phase flow

Denoting $\Lambda^\alpha = \frac{k_r^\alpha(s^\alpha)K}{\mu^\alpha}$ and neglecting gravity

$$\begin{cases} \phi\partial_t s^l - \mathrm{div}\left(\Lambda^l\nabla p^g - \Lambda^l\nabla p_c(s^l)\right) = 0 \\ \quad\quad\;\; - \mathrm{div}\left((\Lambda^g+\Lambda^l)\nabla p^g - \Lambda^l\nabla p_c(s^l)\right) = 0 \end{cases}$$

Primary variable selection

- Good choice (p^g, s^l) : $\Lambda^l p_c'(s^l) < \infty$
- Bad choice (p^g, p_c) : $\frac{\partial s^l}{\partial p_c}$ and Λ^l vanish at dry regions: equation gives $0 \approx 0$



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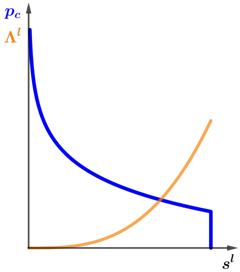

PV selection in the homogeneous two-phase flow

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$$\begin{cases} \phi\left(p_c^{-1}\right)'\partial_t p_c & - & \operatorname{div}\left(\Lambda^l \nabla p^g - \Lambda^l \nabla p_c\right) & = & 0 \\ & - & \operatorname{div}\left((\Lambda^g + \Lambda^l)\nabla p^g - \Lambda^l \nabla p_c\right) & = & 0 \end{cases}$$

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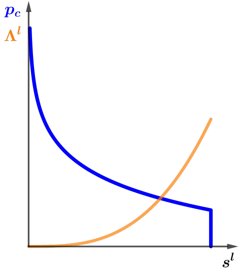

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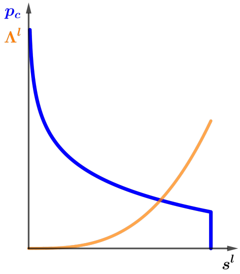
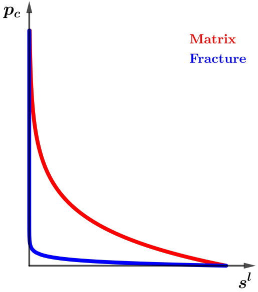
PV selection is more tricky in heterogeneous setting



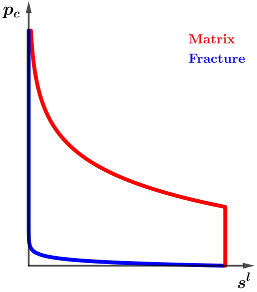
PV selection for two-phase DFM model

Heterogeneous two-phase flow problem

- Multiple switching of the “second” primary variable



Without entry pressure:
$(p^g, s^l_m - s^l_f)$

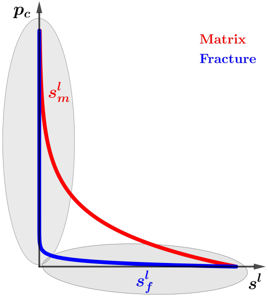


With entry pressure:
$(p^g, s^l_m - p_c - s^l_f)$

PV selection for two-phase DFM model

Heterogeneous two-phase flow problem

- Multiple switching of the "second" primary variable



Without entry pressure:
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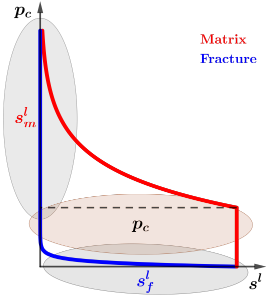
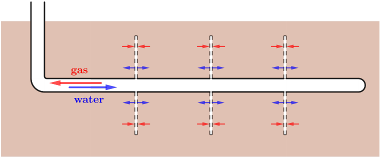
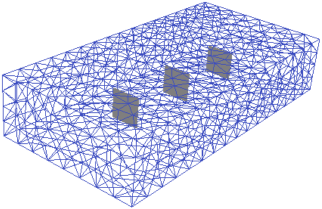


With entry pressure:
$(p^g, s^l_m - p_c - s^l_f)$

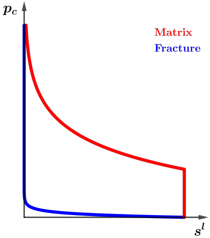
Tight gas recovery test case





Bentsen-Anli model

$$p_{c,i}(s^l) = \begin{cases} [-\infty, p_{ent,i}], & s^l = 1 \\ p_{ent,i} - b_i \log(s^l), & \text{else} \end{cases}$$

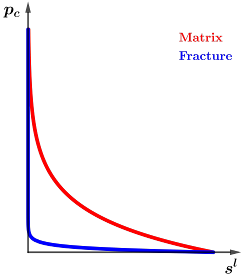


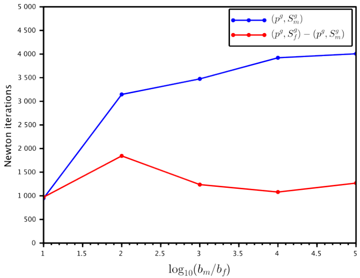
Parameters

- Entry pressure p_{ent}
- Shape parameter b

Performance: (p^g, s^l_m) vs. $(p^g, s^l_m - s^l_f)$

No entry pressure: $p_{c,i} = -b_i \log(s^l)$, $\quad b_m = 10^5$

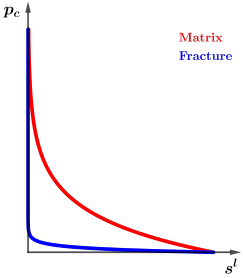


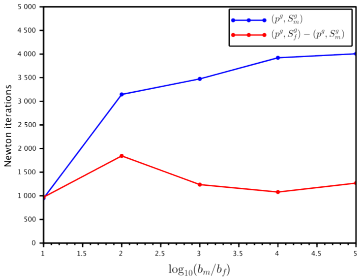


	(p^g, s^l_m)					$(p^g, s^l_m - s^l_f)$				
$\frac{b_m}{b_f}$	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)
10	226	2	4.2	25.9	4 638	226	2	4.3	26.2	5 523
10^2	294	21	10.7	20.1	14 557	246	8	7.5	22.2	9 016
10^3	297	22	11.7	19.7	16 183	225	1	5.5	24.2	6 245
10^4	304	24	12.9	19.8	17 742	225	1	4.8	25.1	5 492
10^5	313	26	12.8	19.6	18 346	235	4	5.4	23.9	6 260
∞	n/a	n/a	n/a	n/a	n/a	235	4	5.3	23.9	6 448

Performance: (p^g, s^l_m) vs. $(p^g, s^l_m - s^l_f)$

No entry pressure: $p_{c,i} = -b_i \log(s^l), \quad b_m = 10^5$





	(p^g, s^l_m)					$(p^g, s^l_m - s^l_f)$				
$\frac{b_m}{b_f}$	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)
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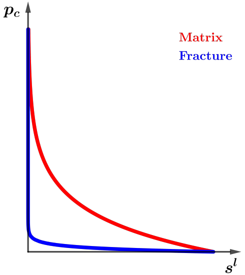
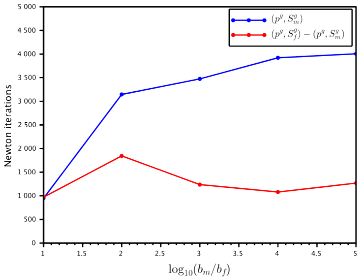
∞	n/a	n/a	n/a	n/a	n/a	235	4	5.3	23.9	6 448

Performance: (p^g, s_m^l) vs. $(p^g, s_m^l - s_f^l)$

No entry pressure: $p_{c,i} = -b_i \log(s^l)$, $\quad b_m = 10^5$





	(p^g, s_m^l)					$(p^g, s_m^l - s_f^l)$				
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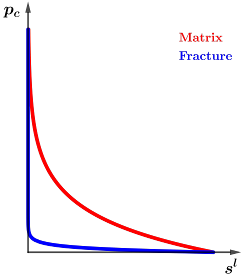

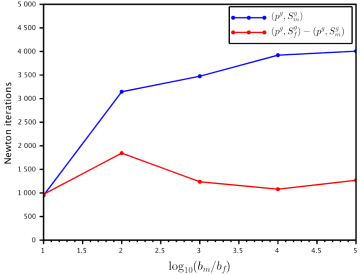

	(p^g, s_m^l)					$(p^g, s_m^l - s_f^l)$				
$\frac{b_m}{b_f}$	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)	N_{dt}	N_{Chop}	N_{Newton}	N_{GMRes}	CPU(s)
10	226	2	4.2	25.9	4 638	226	2	4.3	26.2	5 523
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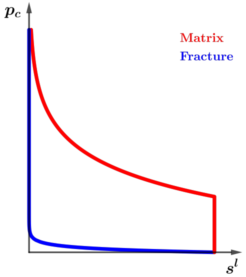
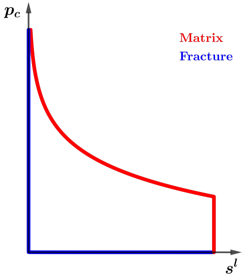
∞	n/a	n/a	n/a	n/a	n/a	235	4	5.3	23.9	6 448

Performance: (p^g, s_f^l) vs. $(p^g, s_m^l - p_c - s_f^l)$





	(p^g, s_f^l)					$(p^g, s_f^l - p_c - s_m^l)$				
$\frac{b_m}{b_f}$	**N**$_{dt}$	**N**$_{Chop}$	**N**$_{Newton}$	**N**$_{GMRes}$	**CPU(s)**	**N**$_{dt}$	**N**$_{Chop}$	**N**$_{Newton}$	**N**$_{GMRes}$	**CPU(s)**
2	221	0	3	29.2	3 937	221	0	3.1	28.9	4 479
10	398	52	9.9	20.2	23 400	262	13	6.8	22.7	10 378
10^2	n/c	n/c	n/c	n/c	n/c	269	14	9.9	20.8	14 185
10^3	n/c	n/c	n/c	n/c	n/c	285	18	8.9	20.1	13 740
10^4	n/c	n/c	n/c	n/c	n/c	242	6	6.9	22.8	9 067
10^5	n/c	n/c	n/c	n/c	n/c	276	16	7.5	21.3	11 516
∞	n/a	n/a	n/a	n/a	n/a	299	22	8.1	19.1	10 770

Conclusions

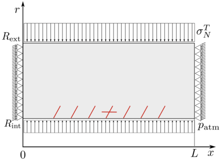
Two-phase DFM models:

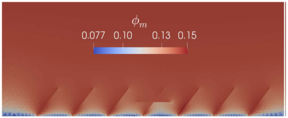
- model selection is delicate;
- even very permeable fractures may act as barriers;
- nonlinear disc. pressure model is very accurate, but less robust.

Robustness of Newton's method can be improved by

- the local variable elimination;
- the appropriate primary variable selection.
- Ongoing work on DD nonlinear preconditioning.

Coupling with mechanical deformation[1,2,3]





[1] Bonaldi, B., Droniou, Masson, 2021

[2] Bonaldi, B., Droniou, Masson, Pasteau, 2021

[3] Bonaldi, Droniou, Masson, Pasteau, 2022