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Interaction between envelope solitons as a model for freak wave formations. Part I: Long time interaction

Didier Clamond, John Grue

Department of Mathematics, University of Oslo, PO Box 1053, Blindern, 0316 Oslo, Norway

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Note presented by Gérard Iooss.

Abstract We are concerned by a special mechanism that can explain the formation of freak waves. We study numerically the long time evolution of a surface gravity wave packet, comparing a fully nonlinear model with Schrödinger-like simplified equations. We observe that the interaction between envelope solitons generates large waves. This is predicted by both models. The fully nonlinear simulations show a qualitative behaviour that differs significantly from the ones preticted by Schrödinger models, however. Indeed, the occurence of freak waves is much more frequent with the fully nonlinear model. This is a consequence of the long-time interaction between envelope solitons, which, in the fully nonlinear model, is totally different from the Schrödinger scenario. The fundamental differences appear for times when the simplified equations cease to be valid. Possible statistical models, based on the latter, should hence under-estimate the probability of freak wave formation. ***To cite this article: D. Clamond, J. Grue, C. R. Mecanique 330 (2002) 575–580.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

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Interaction de solitons enveloppes comme modèle pour la formation de « freak waves ». Partie I : Interaction à long terme

Résumé On s'interresse à un mécanisme particulier pouvant expliquer l'apparition de vagues de grandes amplitudes (freak waves). On étudie numériquement l'évolution à long terme de paquets d'onde de gravité surfaciques. On compare un modèle complètement non linéaire avec des équations simplifiées de type Schrödinger. On observe que l'interaction d'ondes solitaires enveloppes génèrent des vagues de grandes amplitudes. Cela est prédit par tous les modèles. Toutefois, le modèle complètement non linéaire exhibe un comportement à long terme très different des modèles simplifiés. L'apparition de *freak waves* y est beaucoup plus fréquente. C'est une conséquence de l'interaction à long terme de solitons enveloppes, qui est totalement différente de celle prédite par les scénario dérivés des équations de type Schrödinger. Les différences fondamentales apparaissent pour des temps supérieurs aux domaînes de validité des équations simplifiées. D'envisageable modèles statistiques, basés sur ces dernières, devraient donc sous-estimer la probabilité d'apparition de *freak waves*. ***Pour citer cet article : D. Clamond, J. Grue, C. R. Mecanique 330 (2002) 575–580.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

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E-mail addresses: didier@math.uio.no (D. Clamond); johng@math.uio.no (J. Grue).

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L'apparition soudaine, en mer, de vagues de grandes amplitudes – et apparemment anormales – a sévèrement endommagé bateaux et plateformes pétrolières. Ces *freak waves* (ou vagues scélérates) sont ainsi devenues l'objet d'intenses recherches. Les modèles statistiques classiques (gaussiens) sous-estiment largement la probabilité d'apparition de ces vagues. D'autres modèles, basés sur des équations approchées de type Schrödinger, ont donc été proposés. On les compare ici à un modèle complètement non linéaire.

Partant d'une houle progressive exacte, on crée artificiellement un paquet d'onde en la multipliant par une fonction cloche (1). On laisse alors évoluer ce pacquet en utilisant (i) une équation de Schrödinger classique (NLS), (ii) une équation de Dysthe étendue (EDE) [2], (iii) les équations exactes (FNL) [6].

Dans l'exemple considéré, trois solitons enveloppes doivent apparaître, d'après le sénario NLS. C'est en effet ce que l'on obtient avec tous les modèles. Ces solitons interagissent, créant des vagues de grandes amplitudes. On a donc là un mécanisme expliquant les formations de ces vagues.

Avec NLS les trois solitons restent « attachés » et interagissent faiblement, les *freak waves* apparaissent donc relativement rarements. Avec EDE deux petits solitons interagissent faiblement en queue de train, tandis qu'un grand soliton s'échappe en tête, les *freak waves* apparaissant aussi rarement. Quant à lui, le modèle FNL montre que deux solitons interagissent fortement en tête de train, faisant fréquemment apparaître des *freak waves*. En particulier, il arrive que le petit soliton « précède » le grand. C'est un comportement non prévu par les modèles simplifiés. De plus, les solitons finissent par se séparer.

L'interaction de solitons enveloppes fournit ainsi une explication satisfaisante de la formation de *freak waves*. Les modèles simplifiés ne semblent, en revanche, pas capable de décrire correctement ces interactions, du moins sur le long terme. Ils sous-estiment aussi la probabilité d'apparition des *freak waves*.

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1. Introduction

The sudden appearence of large – and apparently abnormal – waves at sea, has caused severe damage to ships and oil platforms. These so-called freak waves (or rogue waves) have become the subject of intense scientific investigations within the last years. Statistical wave models – based on second-order short wave theory (Gaussian waves) – are widely employed by engineers for predicting extreme sea conditions. However, observations have shown that this approach considerably underestimates the probability of large wave events. Actually, with these statistical models, large waves should (almost) never appear. Other models have then to be employed.

The famous nonlinear Schrödinger equation (NLS) is a widely used model for investigating the evolution of surface gravity waves with narrow-banded spectra. A more accurate equation has been proposed by Dysthe [1], and further refined by Trulsen et al. [2]. These weakly nonlinear wave models can be solved very easily numerically, and even analytically for NLS. These models have then be proposed, aiming at understanding formation of freak waves (in deep water). In particular, Osborne et al. [3] suggest the formation of breather as model for freak waves, while Kharif et al. [4] consider nonlinear focussing. Onorato et al. [5] investigated radom sea states. The assumptions of narrow-banded spectra and weak nonlinearities put restrictions on the applicability of such models, however.

Recently, we proposed a fast method for computing fully nonlinear gravity wave equations [6]. This (pseudo-spectral) method is equivalent to the resolution of the complete Zakharov equations. This scheme does not require any smoothing, regridding or other artificial numerical stabilization, that is a very important feature for the validity of long time simulations.

In this work, we study the evolution of a localized wave packet, that evolves creating envelope solitons. (This simulation differs from the two Stokes waves interaction studied by Banner and Tian [7].) We compare fully nonlinear equations (FNE) with NLS and extended Dysthe's equation (EDE) [2]. The interaction between the generated solitons produce large waves, which is another mechanism explaining freak wave formations. This phenomenon is predicted by all the models employed. However, the FNE predicts a long time interaction that is not expected from the simplified models. In a forthcomming paper, we shall describe other highly-nonlinear phenomenae of a freaking wave.

2. Numerical experiments

We investigate the evolution of a localized long wave packet. First, we compute [8] an exact steady Stokes wave, with wavenumber k_0 and amplitude a (a is half the total wave height). The surface elevation and the tangential velocity at the surface are then multiplied by the 'bell' function

$$\operatorname{sech}\left[\varepsilon\sqrt{2}ak_0^2(x-x_0)\right] \tag{1}$$

where the parameter ε determines the length of the packet. The case $\varepsilon = 1$ corresponds to an exact NLS soliton. This problem has been studied analytically by Satsuma and Yajima [9] using NLS, and numerically by Lo and Mei [10] using Dysthe's equation [1]. We complete their works in comparing simulations using the fully nonlinear equations [6] with the NLS and the extended Dysthe's equation [2].

We consider here a wave packet with $ak_0 = 0.091$ and $\varepsilon = 0.263$. The computational domain involves 128 wavelengths, and the carrier wave is discretized over 32 nodes per wavelength. This means that all harmonics up to the 15th are resolved, and that 128 Fourier modes are included in the spectral band $[k_0 - \frac{1}{2}k_0; k_0 + \frac{1}{2}k_0]$.

According to predictions by NLS, three solitons should be formed, and in addition, some dispersive tails. This is indeed demonstrated by all the models. The initially long group splits, after some time, into solitons which become interacting (see Fig. 1). This interaction is the focus here.

Large waves (i.e. large crest or deep through), up to three times the initial maximum elevation, are then formed. These freak waves oscillate in time for a while. After a period, the 'freaking' stops, also for a while. Then it starts again, and stops, and restarts, etc. To understand this recurrent phenomenon, we consider the wave envelope (of the fundamental wavelength).

With the fully nonlinear simulations, two solitons appear in the front of the train (see Fig. 2). The flow is highly unsteady. A rapid exchange of energy takes places. Large and frequent waves are thus formed. More surprisingly, the small soliton can 'pass ahead' of the larger one (Fig. 2(b), (c)). This is an unexpected behaviour, according to simplified models. Eventually, the three solitons separate (Fig. 2(d)). This is in agreement with the experimental observations (Yuen and Lake [11]).

With the extended Dysthe equation (EDE), the two-soliton appears in the back of the train. Once formed, the solitons are almost steady. A large steady soliton is running ahead. Little energy is exchanged between the solitons. A weak interaction indicates a gentle variation of the height of the two last solitons. As a consequence, no very big waves are formed.

With the classical cubic Schrödinger equation, the three solitons remain 'attached' together. This three-soliton then propagates steadily forever (Fig. 2(b)), with a symmetric profile and little interaction. NLS under-estimates the occurrence of freak wave formation, as does EDE.

For small times however, the three models match. In particular, the first large wave event is rather well predicted in both time and amplitude (Fig. 2(a)). This is logical since EDE is valid while $(ak_0)^3\omega_0 t \leqslant \mathrm{O}(1)$ and NLS while $(ak_0)^2\omega_0 t \leqslant \mathrm{O}(1)$.

3. Conclusion

We have performed, for the first time, a fully nonlinear simulation of the long time evolution of a wave packet. We have compared this simulation with simplified models of Schrödinger type. We have found that the interactions between envelope solitons provide a satisfactory explanation for the formation of freak waves. Although the simplified wave equations can model the evolution and interaction of solitons, their long time evolution is significantly inaccurate. The behaviour of soliton interaction is much more 'quiet'

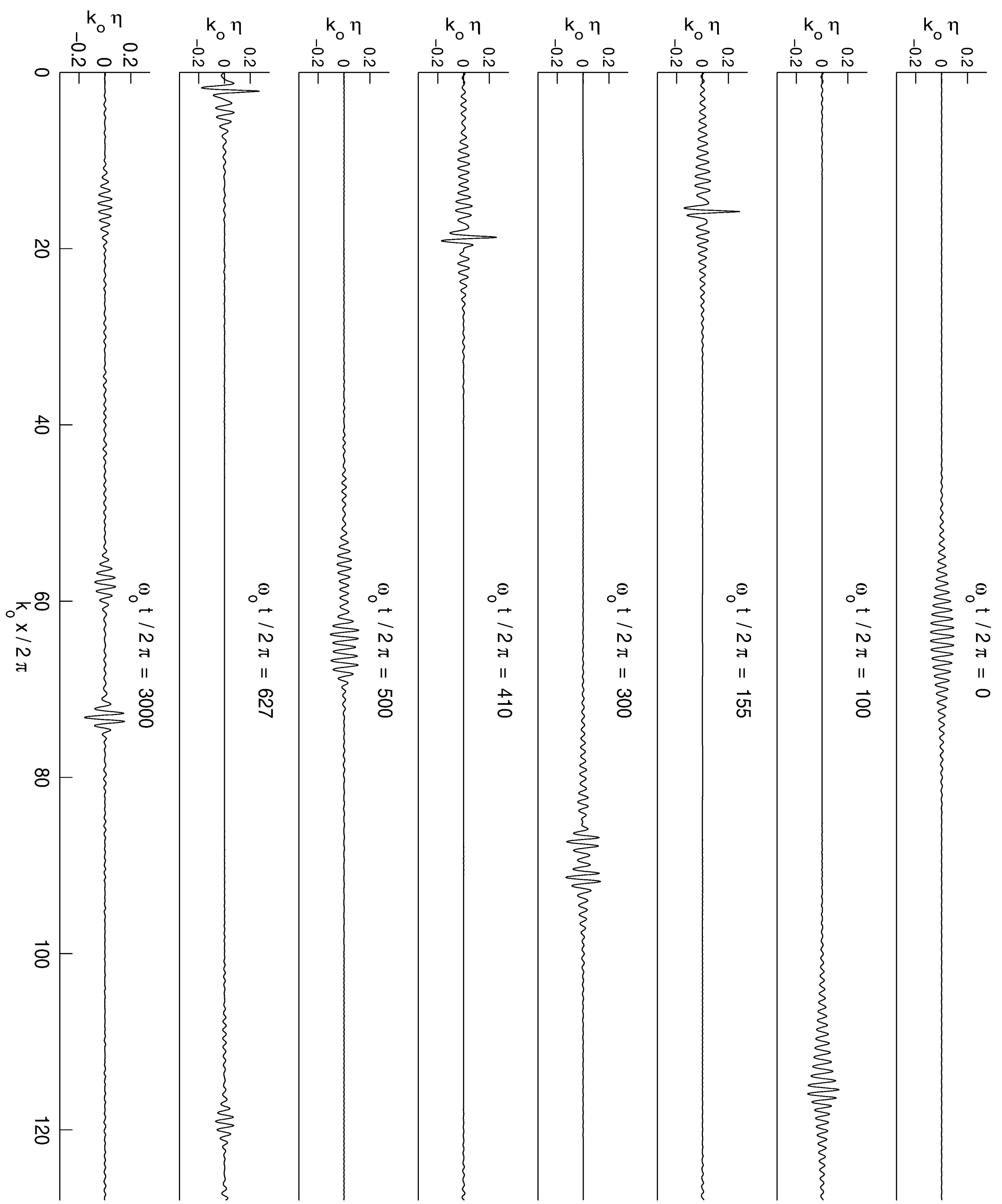

Figure 1. Surface elevation at different times. Fully nonlinear simulations.

Figure 1. *Élévation de la surface à différents instants. Simulations complètement non linéaires.*

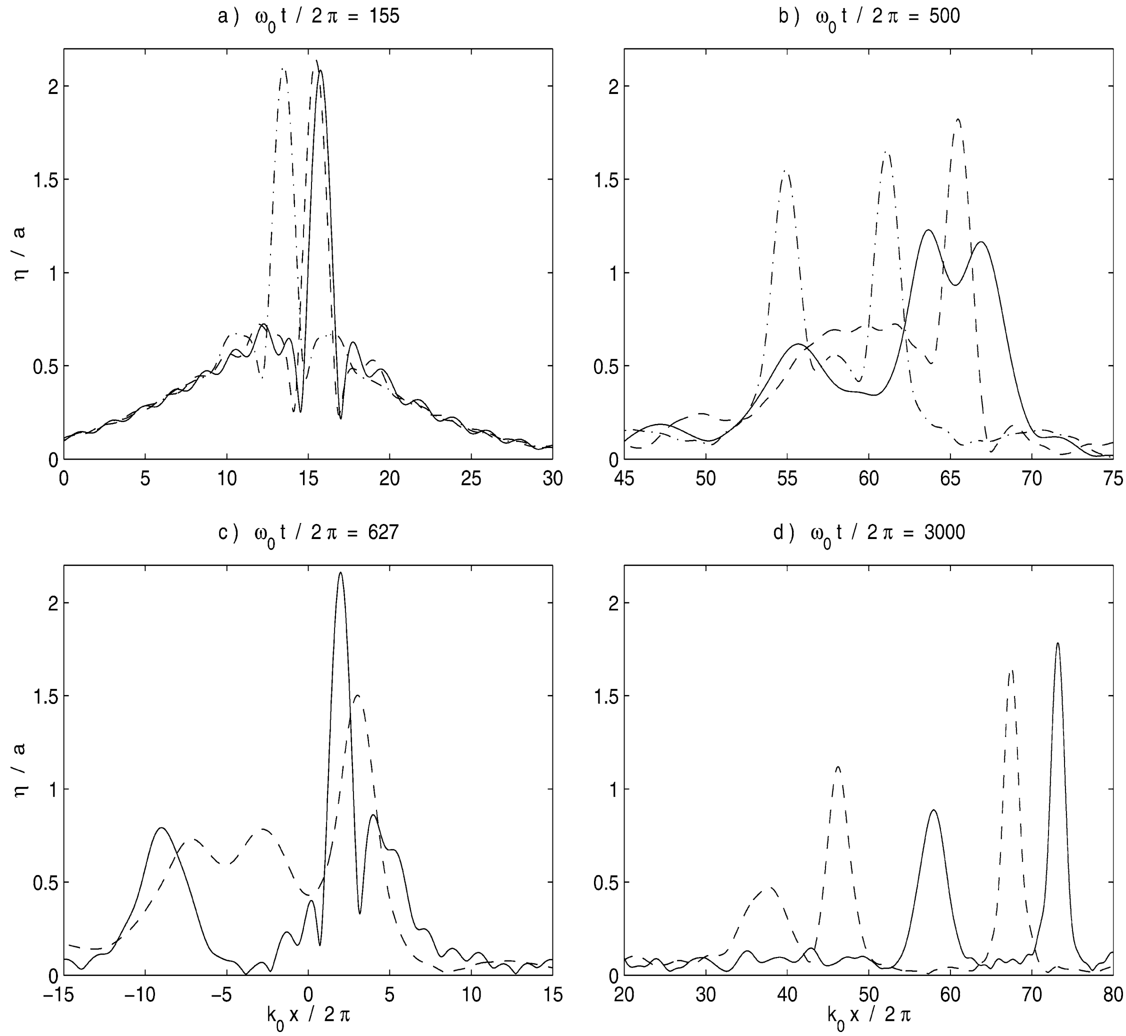

Figure 2. Surface's envelope at different times. — FNE, – – EDE, – · NLS.

Figure 2. *Enveloppe de la surface à différents instants.*

than the one observed in the fully nonlinear simulations. As a consequence, simplified models highly underestimate the occurence of freak waves. Hence, a possible statistical model for freak wave predictions, based on Schrödinger-like equations, may significantly underestimate their probability.

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