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# A new model for the blown film process

## *Un nouveau modèle du procédé de soufflage de gaine*

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ABSTRACT

Polymer films are generally manufactured by film blowing. In this process the polymer (a polyethylene for example) is molten in a screw extruder and forced into a tubular die (typical dimensions are several decimeters in diameter and about one mm in thickness). At extrusion, it forms a liquid tube which is simultaneously drawn in the vertical direction by nip rolls, inflated by an internal pressure and cooled by external air rings. Typical dimensions of the bubble at take up are 1 m or more in diameter and several 10 μm in thickness. From a mechanical point of view, it is an extensional thin layer flow. Readers not familiar with this process will find easily pictures and schematic descriptions with a web research using keywords *blown film extrusion*. In order to simplify, it is assumed that the temperature profile is known and that the molten polymer behaves as a Newtonian fluid. This crude rheological behavior allows to capture qualitatively an important part of observed phenomena.

The classical model introduced by Pearson and Petrie in 1970 is based on three hypothesis: the polymer flow in air is steady and axisymmetric and the film is thin. It uses a tangent frame affixed to the membrane to describe kinematics and to compute stress and strain tensors. In this model the balance equations are written using a stretching force and a curvature equation. It results in a nonlinear system of differential equations for velocity, thickness, radius and stress components according to distance $z$ to extrusion. Solution is then computed using a tedious shooting method to determine force and internal pressure. For stability reasons this system of equations is solved from take up to extrusion.

In this Note we derive also balance equations according to a curvilinear abscissa $s$. It allows us to derive a coupled system of equations for velocity and geometry according to radius $r$. This strategy which is classically used to determine surfaces of revolution with given mean curvature is rather disconcerting to compute a stretching flow in $z$ direction. However it leads to a model constituted of two coupled equations:

- Velocity is then solution of a Dirichlet boundary value problem of order two easily solved using a finite elements method;
- The profile curve of the bubble is solution of a highly nonlinear differential equation of order one leading to a singular integral. Internal pressure appears as a parameter allowing to impose an additional boundary condition.

Despite the fact that the computation of the solution remains technical, this new model seems more natural from a mathematical point of view as it leads, on the one hand, to an elliptic equation to compute velocity, and on the other hand, to an equation of a classical type in differential geometry to determine the generatrix.



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R É S U M É

Le procédé de soufflage de gaine est un procédé industriel largement utilisé pour la fabrication des films synthétiques. Le principe est le suivant :

– Le polymère fondu (un polyéthlène par exemple) est extrudé à travers une filière annulaire ;
– Après extrusion le polymère forme un cylindre liquide qui est bi-étiré : une vitesse de bobinage supérieure à la vitesse moyenne d'extrusion impose un étirage vertical et une surpression interne permet un étirage radial ;
– Le film est refroidi par une soufflerie d'air puis la bulle est pincée entre deux rouleaux (ce qui ferme la bulle et permet la surpression interne) et enfin mise à plat.

Le lecteur non familier trouvera aisément des photographies et schémas du dispositif en utilisant un moteur de recherche et les mots clés *blown film extrusion*. Du point de vue de la mécanique des milieux continus, il s'agit d'un écoulement essentiellement élongationnel (le cisaillement est négligeable) en couche mince. Pour simplifier l'exposé il est supposé que le profil de température est connu et que le comportement est newtonien, avec une viscosité dépendante de la température selon une loi d'Arrhenius, par exemple.
Le modèle classiquement utilisé est celui de Pearson et Petrie (1970) reposant sur trois hypotheses fondamentales : l'ecoulement du polymère est stationnaire, axisymétrique, et le film est mince. Ce modèle utilise un repère lié à la membrane pour écrire les tenseurs des vitesses de déformation et des contraintes. L'équation d'équilibre est écrite en utilisant la force d'étirage et une équation de courbure. Il résulte de l'ensemble de ces équations un système différentiel en $z$ (qui représente la distance à l'extrusion sur l'axe de symétrie) pour la vitesse, l'épaisseur, le rayon et les composantes du tenseur des contraintes. Le calcul effectif de la forme de la bulle se fait alors par une délicate méthode de tir permettant l'ajustement de la force d'étirage et de la surpression interne. Le système différentiel est en général intégré du haut vers le bas pour des raisons de stabilité.
Dans cet article l'équation d'équilibre dans la direction de l'étirage est écrite selon l'abcisse curviligne $s$. Cela permet ensuite d'obtenir un système couplé d'équations pour la vitesse et la géométrie utilisant la variable $r$. Cette méthode qui est classiquement utilisée pour calculer les surfaces de révolution de courbure moyenne donnée est nouvelle dans ce domaine. Le modèle obtenu est constituée de deux équations couplées pour la vitesse et le profil de la bulle :

– La vitesse est solution d'un problème de Dirichlet avec conditions aux extrémités de l'intervalle d'ordre 2 qui est facilement résolue par une méthode de type éléments finis ;
– La génératrice de la bulle est solution d'une équation différentielle fortement non linéaire d'ordre 1 conduisant à une intégrale singulière. La surpression interne apparaît alors comme un paramètre additionnel permettant de fixer cette courbe à chaque extrémité de l'intervalle.

En dépit du fait que la résolution du problème demeure technique, ce nouveau modèle semble mathématiquement plus naturel puisqu'il conduit d'une part à une équation elliptique avec conditions aux bords pour la vitesse et d'autre part à une équation de type classique en géométrie dans l'étude des surfaces de révolution.


## 1. State of the art

### 1.1. The blown film process

The blown film process is largely used to manufacture polymer films. The polymer (a polyethylene for example) is molten in a screw extruder and forced into a tubular die (typical dimensions are several decimeters in diameter and about one mm in thickness). At extrusion, it forms a liquid tube which is simultaneously drawn in the vertical direction by nip rolls, inflated by an internal pressure and cooled by external air rings. Typical dimensions of the bubble at take up are 1 m or more in diameter and several 10 μm in thickness. Fig. 1 presents a picture of the bubble. From a mechanical point of view, it is an extensional thin layer flow. Readers not familiar with this process will find easily pictures and schematic descriptions with a web research using keywords *blown film extrusion*.

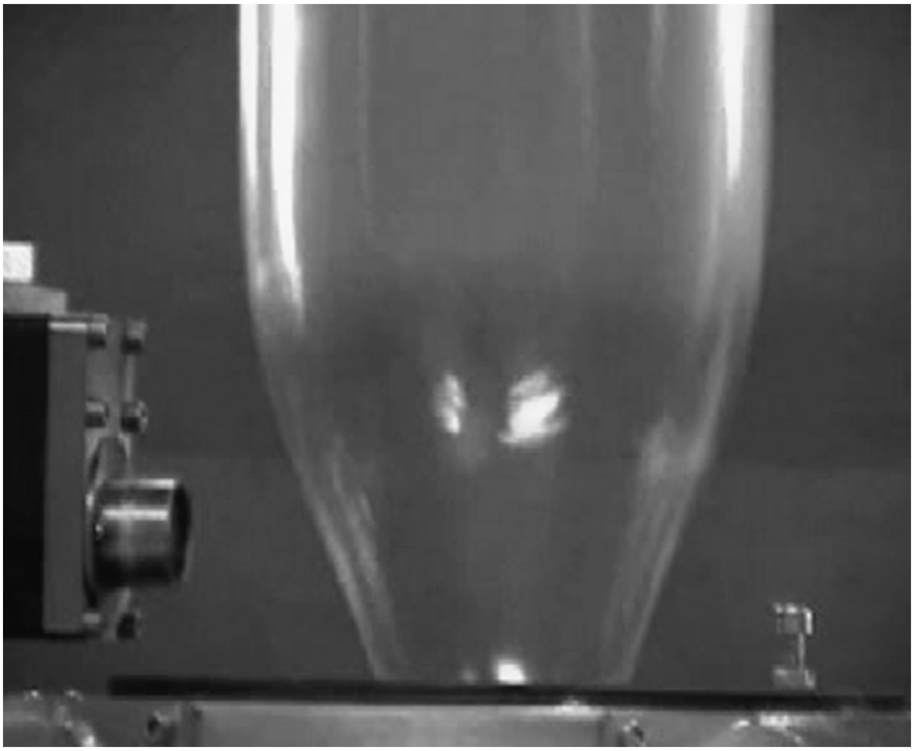

**Fig. 1.** Picture of the blown film process.

### 1.2. The Pearson and Petrie membrane model

Due to its economical importance for polymer film production, the blown film extrusion process has been largely studied experimentally and theoretically (see [1] for a general overview). In these models, variations of the physical variables (velocity, temperature, stresses, ...) across the thin film are neglected. All these models are in the limit of very thin films and issued of papers of Pearson [2] and Pearson and Petrie [3]. Equations involve the continuity equations, momentum equations in the axial and circumferential directions, and some type of constitutive relations. A Newtonian model with a temperature dependent viscosity was considered for example in [3] and [4]. Even if more complex rheological behavior (shear thinning, viscoelasticity or polymer-crystallization effect) was considered in further works, the basic mechanical ideas remains the same, despite possibles technical difficulties. In [3] the bubble is assumed to be steady and axisymmetric. Variable $z$ denotes on vertical symmetry axis the distance to extrusion and the bubble is described as a function $r = R(z)$. The mechanical description uses a local frame tangent to the profile curve. The velocity $V(z)$ is tangent to the generatrix, and components of the rate of strain tensor are written using derivatives of the radius, velocity and thickness according to variable $z$. Mass conservation and Newtonian constitutive equations are used to compute stress tensor components in the tangent and normal direction to the generatrix as well as in the azimuthal direction. As internal pressure in the bubble is small compared to extensional part of the stress tensor, the normal component is assumed to be zero. Normal component of the balance equations gives an equation involving curvature radii and remaining stress tensor components. The missing part of equilibrium equations is written using the stretching force in axial direction (this point will be discussed and an other method will be proposed). The whole system of differential equations according to variable $z$ is solved using a technical and tedious shooting method.

This membrane model can be derived in a more rigorous way from the Stokes system of equations for a Newtonian fluid (inertia and gravity can be neglected). Mass conservation, Newtonian behavior and balance equations are then written in the flow domain as well as stress and kinematical equations on the internal and external interfaces with air. A small parameter $\epsilon$ defined as the ratio of thickness of the film to radius of the bubble at extrusion is introduced using convenient dimensionless variables. The membrane model is then obtained as a limit system of equations for $\epsilon \to 0$.

### 1.3. Other studies

This article is not concerned with an important part of the literature on blown film process devoted to the stability. A periodic instability, referred as draw resonance instability occurring at high stretching ratio (in axial or radial directions) limits production. This instability understood mathematically as a Hopf bifurcation is encountered in most of extensional polymer flows with free surface. The stable zone in the space of processing parameters is then determined as the domain

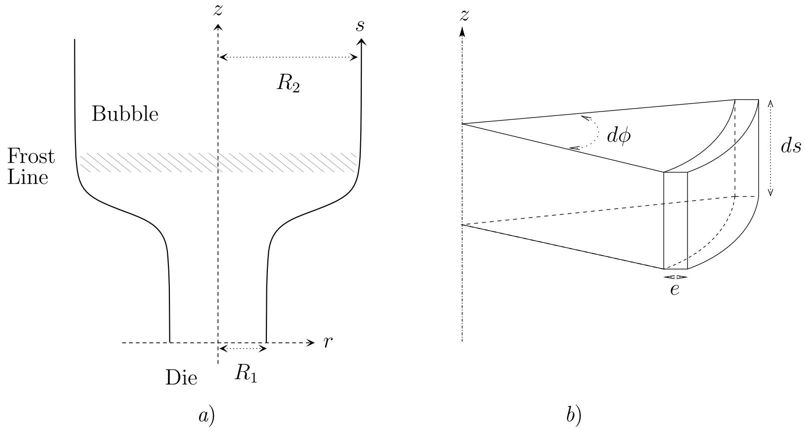


**Fig. 2.** Definition sketch.

where real parts of all eigenvalues are negative. These computations required obviously a transient model. Physical properties (crystallinity, transparency or gloss, residual stresses) of the solid polymer film have been also extensively studied in reason of their practical importance for packaging industry.

## 2. The membrane model

### 2.1. Geometry

As the membrane is a surface of revolution, a system of cylindrical coordinates $(r, \phi, z)$ is used and vectors are written in the frame $(\vec{e}_r, \vec{e}_\phi, \vec{e}_z)$ (see Fig. 2a). If variable $s$ denotes curvilinear abscissa on the generatrix, this curve is defined by equations $r = R(s)$ and $z = Z(s)$ with $0 \leqslant s \leqslant L$. The length $L$ is assumed to be large enough to reach solidification of the film.

Let us note $\vec{t}$ the tangent vector to the generatrix and $\alpha$ the angle $(\vec{e}_r, \vec{t})$. We have obviously:

$$\cos\big(\alpha(s)\big) = \frac{\mathrm{d}R}{\mathrm{d}s}; \qquad \sin\big(\alpha(s)\big) = \frac{\mathrm{d}Z}{\mathrm{d}s} \quad \text{and} \quad \tan\big(\alpha(s)\big) = \frac{\mathrm{d}Z}{\mathrm{d}r} \tag{1}$$

In practical conditions of the blown film process, we have $0 < \alpha < \frac{\pi}{2}$ in the liquid part of the bubble and $\alpha = \frac{\pi}{2}$ at solidification. Tangent and normal vectors $\vec{t}(s)$ and $\vec{n}(s)$ can be written as:

$$\vec{t}(s) = \cos\big(\alpha(s)\big)\vec{e}_r + \sin\big(\alpha(s)\big)\vec{e}_z$$
$$\vec{n}(s) = -\sin\big(\alpha(s)\big)\vec{e}_r + \cos\big(\alpha(s)\big)\vec{e}_z$$

and hence:

$$\frac{\mathrm{d}\vec{t}}{\mathrm{d}s} = \frac{\mathrm{d}\alpha}{\mathrm{d}s}\vec{n} \quad \text{with } \frac{\mathrm{d}\alpha}{\mathrm{d}s} = \frac{1}{\mathcal{R}_1}$$

Quantity $\mathcal{R}_1$ is the curvature radius of the plane curve $z = Z(r)$ in the frame $(\vec{e}_r, \vec{e}_z)$. Let us note $e(s)$ the thickness of the membrane. This thickness is small compared to others dimensions and the model described in the following is a limit case for $\frac{e}{\mathcal{R}_1} \to 0$.

### 2.2. Kinematics

The mean velocity $\vec{V}$ is tangent to the generatrix:

$$\vec{V}(s) = V(s)\vec{t}(s)$$

It means that shearing is neglected. The rate of strain tensor is then written using derivatives of functions $V(s)$, $e(s)$ and $R(s)$ (see for example [1]):

$$\dot{\boldsymbol{\epsilon}} = \dot{\epsilon}_t \vec{t} \otimes \vec{t} + \dot{\epsilon}_n \vec{n} \otimes \vec{n} + \dot{\epsilon}_\phi \vec{e}_\phi \otimes \vec{e}_\phi$$

with

$$\dot{\epsilon}_t = \frac{\mathrm{d}V}{\mathrm{d}s}; \qquad \dot{\epsilon}_n = \frac{V}{e}\frac{\mathrm{d}e}{\mathrm{d}s}; \qquad \dot{\epsilon}_\phi = \frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}s} \tag{2}$$

For a steady bubble the trace of rate of strain tensor $\dot{\boldsymbol{\epsilon}}$ is zero because of mass conservation ($Q$ is the flow rate):

$$\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}s}+\frac{1}{e}\frac{\mathrm{d}e}{\mathrm{d}s}+\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}s}=0 \quad \Leftrightarrow \quad 2\pi ReV=Q \tag{3}$$

*2.3. Behavior*

As the polymer is assumed to behave as a Newtonian fluid, we have:

$$\boldsymbol{\sigma}=\sigma_t\vec{t}\otimes\vec{t}+\sigma_n\vec{n}\otimes\vec{n}+\sigma_\phi\vec{e}_\phi\otimes\vec{e}_\phi$$

with

$$\sigma_t=2\eta\frac{\mathrm{d}V}{\mathrm{d}s}-p; \qquad \sigma_n=2\eta\frac{V}{e}\frac{\mathrm{d}e}{\mathrm{d}s}-p; \qquad \sigma_\phi=2\eta\frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}s}-p$$

Component $\sigma_n$ is small compared to $\sigma_t$ and $\sigma_\phi$. It can be proven, using an expansion according to the small parameter $\epsilon$, that $\sigma_n$ is of order $\epsilon$ while $\sigma_t$ and $\sigma_\phi$ are of order 1 (see [5] and [6]). Using $\sigma_n=0$ we have $p=2\eta\frac{V}{e}\frac{de}{ds}$ and using (3) we obtain finally:

$$\sigma_t=2\eta\left(2\frac{\mathrm{d}V}{\mathrm{d}s}+\frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}s}\right); \qquad \sigma_\phi=2\eta\left(\frac{\mathrm{d}V}{\mathrm{d}s}+2\frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}s}\right) \tag{4}$$

If the fluid behaves as a Maxwell fluid, Eqs. (4) are more complex (see [7]). In order to simplify, the temperature $T$ is considered as a known function of variable $z$ (or $s$):

$$\eta=\eta\big(T(z)\big)$$

*2.4. Balance equations*

The static equilibrium is written on a small element of surface $\mathrm{d}s\times R\,\mathrm{d}\phi$ and thickness $e$ (see Fig. 2b). The sum of internal forces acting on the boundaries of this small element can be splitted in the following form:

$$\begin{aligned}&\big[e(s+\mathrm{d}s)R(s+\mathrm{d}s)\sigma_t(s+\mathrm{d}s)\vec{t}(s+\mathrm{d}s)-e(s)R(s)\sigma_t(s)\vec{t}(s)\big]\,\mathrm{d}\phi+\big[\sigma_\phi(s)\vec{e}_\phi(\phi+\mathrm{d}\phi)-\sigma_\phi(s)\vec{e}_\phi(\phi)\big]e(s)\,\mathrm{d}s\\&\quad=\left[\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t\vec{t})+e\sigma_\phi\frac{\mathrm{d}}{\mathrm{d}\phi}(\vec{e}_\phi)\right]\mathrm{d}s\,\mathrm{d}\phi=\left[\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t)\vec{t}+e\sigma_t\frac{R}{\mathcal{R}_1}\vec{n}-e\sigma_\phi\vec{e}_r\right]\mathrm{d}s\,\mathrm{d}\phi\end{aligned}$$

As gravity forces as neglected compared to stretching forces, these internal forces are in equilibrium with the external force due to internal pressure $\Delta p$. The balance equation is then:

$$\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t)\vec{t}+e\sigma_t\frac{R}{\mathcal{R}_1}\vec{n}-e\sigma_\phi\vec{e}_r-R\Delta p\vec{n}=0$$

An orthogonal projection of this equation onto the normal $\vec{n}$ gives the hoop stress-like equation ($\mathcal{R}_2$ denotes the other curvature radius of the revolution surface):

$$\frac{e\sigma_t}{\mathcal{R}_1}+\frac{e\sigma_\phi}{\mathcal{R}_2}=\Delta p \quad \text{with } \frac{1}{\mathcal{R}_1}=\frac{\mathrm{d}\alpha}{\mathrm{d}s}; \quad \frac{1}{\mathcal{R}_2}=\frac{\sin(\alpha)}{R} \tag{5}$$

The second balance equation is obtained with an orthogonal projection onto tangent $\vec{t}$ (this equation is not classical):

$$\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t)-e\cos(\alpha)\sigma_\phi=0 \tag{6}$$

*2.5. Boundaries conditions*

Thickness, radius and velocity are known at extrusion ($s=0$):

$$V(0)=V_1; \qquad R(0)=R_1; \qquad e(0)=e_1 \tag{7}$$

as well as velocity at a large enough distance to extrusion (the film is solidified at take up, $s=L$):

$$V(L)=V_2 \tag{8}$$

The internal pressure $\Delta p$ is adjusted to impose a given value of radius at take up:

$$R(L)=R_2 \tag{9}$$

Eqs. (3)–(6) with boundaries conditions (7)–(9) describe the blown film process. Important dimensionless numbers are the Draw Ratio (*DR*) and the Blow Up Ratio (*BUR*):

$$DR = \frac{V_2}{V_1} \qquad BUR = \frac{R_2}{R_1}$$

## 3. The Pearson–Petrie model

### 3.1. A first integral for the system

Let us define the force $F(s)$ as:

$$F(s) = \pi\left[2e(s)R(s)\sigma_t(s)\sin\bigl(\alpha(s)\bigr) - \Delta p\bigl(\bigl(R(s)\bigr)^2 - R_1^2\bigr)\right] \tag{10}$$

Let us prove that $F$ is a constant:

$$\begin{aligned}
\frac{1}{2\pi}\frac{\mathrm{d}F}{\mathrm{d}s} &= \sin(\alpha)\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t) + \cos(\alpha)e\sigma_t\frac{R}{\mathcal{R}_1} - \Delta pR\frac{\mathrm{d}R}{\mathrm{d}s} \\
&= \sin(\alpha)\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t) + \cos(\alpha)\left(e\sigma_t\frac{R}{\mathcal{R}_1} - R\Delta p\right) \quad \text{using } \frac{\mathrm{d}R}{\mathrm{d}s} = \cos(\alpha) \\
&= \sin(\alpha)\left(\frac{\mathrm{d}}{\mathrm{d}s}(eR\sigma_t) - e\cos(\alpha)\sigma_\phi\right) \quad \text{using (5)} \\
&= 0 \quad \text{using (6)}
\end{aligned}$$

This constant is the stretching force at extrusion.

### 3.2. The Pearson and Petrie model

It is a system of differential or algebraic equations ([3]) for functions $V$, $R$, $e$ and angle $\theta = \frac{\pi}{2} - \alpha$ according to variable $z$. It corresponds to Eqs. (1), (3), (4), (5) and (10):

$$\mathrm{tg}(\theta) = \frac{\mathrm{d}R}{\mathrm{d}z} \tag{11a}$$

$$Q = 2\pi eVR \tag{11b}$$

$$\sigma_t = 2\eta\cos(\theta)\left(2\frac{\mathrm{d}V}{\mathrm{d}z} + \frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}z}\right) \tag{11c}$$

$$\sigma_\phi = 2\eta\cos(\theta)\left(\frac{\mathrm{d}V}{\mathrm{d}z} + 2\frac{V}{R}\frac{\mathrm{d}R}{\mathrm{d}z}\right) \tag{11d}$$

$$2\pi Re\sigma_t\cos(\theta) = F_1 + \pi\Delta p\left(R^2 - R_1^2\right) \tag{11e}$$

$$\frac{\sigma_t}{\mathcal{R}_1} + \frac{\sigma_\phi}{\mathcal{R}_2} = \frac{\Delta p}{e} \quad \text{with } \frac{1}{\mathcal{R}_1} = -\frac{\mathrm{d}^2R}{\mathrm{d}z^2}\bigl(\cos(\theta)\bigr)^3 \quad \text{and} \quad \frac{1}{\mathcal{R}_2} = \frac{\cos(\theta)}{R} \tag{11f}$$

Let be $H = Z(L)$. This set of equations is associated with boundary conditions in $z = 0$ and $z = H$ deduced from (7)–(9). It allows to compute $V$, $R$, $e$ and $\frac{\mathrm{d}R}{\mathrm{d}z}(z)$ as the solution of a differential equation from $z = H$ to $z = 0$ and constants $\Delta p$ and $F$ are used as shooting variables to satisfy boundaries conditions $R(0) = R_1$ and $V(0) = V_1$. Let us recall that the height $H$ is chosen large enough to have a solidified film:

$$\eta\bigl(T(0)\bigr) \ll \eta\bigl(T(H)\bigr)$$

## 4. Equations according to variable *r*

In the following the system of equations (3)–(6) is written as a system of differential equations according to variable $r$ in the interval $[R_1, R_2]$. This strategy is classical to compute minimal surfaces of revolution [8].

### 4.1. Mass conservation

Eq. (3) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}r}(rVe) = 0 \tag{12}$$

*4.2. Behavior*

Using $\mathrm{d}r = \cos(\alpha)\,\mathrm{d}s$, Eq. (4) becomes:

$$\sigma_t = 2\eta\cos(\alpha)\left(2\frac{\mathrm{d}V}{\mathrm{d}r} + \frac{V}{r}\right); \qquad \sigma_\phi = 2\eta\cos(\alpha)\left(\frac{\mathrm{d}V}{\mathrm{d}r} + 2\frac{V}{r}\right) \tag{13}$$

*4.3. The velocity equation*

The stretching component of the balance equation (6) becomes:

$$-\frac{\mathrm{d}}{\mathrm{d}r}(er\sigma_t) + e\sigma_\phi = 0$$

Using (13), it appears that velocity $V$ is solution of a Dirichlet boundary value problem of order 2:

$$\begin{cases} -\dfrac{\mathrm{d}}{\mathrm{d}r}\left(qr\left(2\dfrac{\mathrm{d}V}{\mathrm{d}r} + \dfrac{V}{r}\right)\right) + q\left(\dfrac{\mathrm{d}V}{\mathrm{d}r} + 2\dfrac{V}{r}\right) = 0 \\ V(R_1) = V_1;\ V(R_2) = V_2 \end{cases} \tag{14}$$

In this equation $q$ is defined as $q(r) = 2\eta(T(r))\cos(\alpha(r))e(r)$. Let us notice that $\eta(T(R_2))$ is large while $\cos(\alpha(R_2))$ is small but $q(R_2)$ has a finite non-zero value. Furthermore Eq. (14) has a variational formulation. Multiplying both sides of (14) by a test function $W$ satisfying $W(R_1) = W(R_2) = 0$ and integrating by parts on interval $[R_1, R_2]$, we obtain:

$$a(V, W) = \int_{R_1}^{R_2} q\left(2r\frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}W}{\mathrm{d}r} + V\frac{\mathrm{d}W}{\mathrm{d}r} + \frac{\mathrm{d}V}{\mathrm{d}r}W + \frac{2}{r}VW\right)\mathrm{d}r$$

It is interesting to notice that this symmetric bilinear form $a$ is coercive if the minimum of $q$ on interval $[R_1, R_2]$ is non-zero:

$$\begin{aligned} a(V, V) &= \int_{R_1}^{R_2} q\left(2r\left(\frac{\mathrm{d}V}{\mathrm{d}r}\right)^2 + 2V\frac{\mathrm{d}W}{\mathrm{d}r} + \frac{2}{r}V^2\right)\mathrm{d}r \\ &= \int_{R_1}^{R_2} q\left(r\left(\frac{\mathrm{d}V}{\mathrm{d}r}\right)^2 + \frac{1}{r}V^2 + \left(\sqrt{r}\frac{\mathrm{d}V}{\mathrm{d}r} + \frac{V}{\sqrt{r}}\right)^2\right)\mathrm{d}r \\ &\geqslant \beta\|V\|_1^2 \end{aligned}$$

where $\|V\|_1^2$ is the classical $H^1$ norm. It means that Eq. (14) is easily solved by a finite elements method if the function $q(r)$ is known.

*4.4. Geometrical equation*

So far, an equation for the profile curve is missing. Expressing Eq. (10) according to variable $r$ as a constant:

$$F(r) = \pi\left[2re(r)\sigma_t(r)\sin(\alpha(r)) - \Delta p\left(r^2 - R_1^2\right)\right] = F(R_2)$$

The bubble is tangent to $z$ axis in $r = R_2$ and hence $\sin(\theta(R_2)) = 1$:

$$2re(r)\sigma_t(r)\sin(\alpha(r)) = 2R_2e(R_2)\sigma_t(R_2) - \Delta p\left(R_2^2 - r^2\right)$$

This equation has the form:

$$\sin(\alpha(r)) = A(r) - \Delta pB(r) \quad \text{with } A(r) = \frac{R_2e(R_2)\sigma_t(R_2)}{re(r)\sigma_t(r)} \quad \text{and} \quad B(r) = \frac{R_2^2 - r^2}{2re(r)\sigma_t(r)}$$

Function $Z(r)$ describing the generatrix satisfies the nonlinear differential equation:

$$\frac{Z'}{\sqrt{1 + Z'^2}} = A(r) - \Delta pB(r) \quad \text{with } Z(R_1) = Z_1 \quad \text{and} \quad Z(R_2) = Z_2 \tag{15}$$

The parameter $\Delta p$ is determined to satisfy the second boundary condition. If $Z_{\Delta p}(r)$ is the solution of Eq. (15) satisfying boundary condition $Z_{\Delta p}(R_1) = Z_1$ for a given value of $\Delta p$, the value $Z_{\Delta p}(R_2)$ is a decreasing function of $\Delta p$. It allows to determine $\Delta p$ such that $Z_{\Delta p}(R_2) = Z_2$. However Eq. (15) remains highly nonlinear and solution $Z_{\Delta p}(r)$ does not exist for all values of $\Delta p$.

## 5. Conclusion

This new model is constituted of the differential equations according to variable $r$ (14) and (15) and of the algebraic equation (12). These equations are coupled through expressions (13). Even if convergence remains difficult, this system of equations is more natural from a mathematical point of view. It is expected to be more convenient to describe the liquid part of the bubble from extrusion to the so called freeze-line height where solidification occurs. At the contrary it is not adapted to the study of the physical morphology of the solidified film as this morphology is linked to physical conditions existing in the vertical part of the bubble i.e. at the end of stretching. An other limitation is that, at this point, it is a steady model and as a consequence it is obviously not adapted to study the draw resonance instability inducing large variations of radius and thickness at solidification. However, it seems possible to overcome this difficulty in the future.

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