Here today, gone tomorrow: Can dynamic incentives make microfinance more flexible?

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Abstract

This paper presents a model of microfinance lending to individuals that uses dynamic incentives, in the form of access to additional loans, to discourage borrowers from strategic default, or the unwillingness to repay a loan once a positive outcome is realized. We propose an improvement on contracts currently used by microfinance institutions (MFIs) by endogenizing the default penalty, while constraining the MFI to maintain sustainable lending operations. Furthermore, accounting for the risks that the poor face by including a negative economic shock, we show that under certain circumstances, the punishment for default need not be a lifetime without loans.

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1. Introduction

Researchers are attempting to unravel the reasons behind the success of microfinance institutions (MFIs): they target the poor, the risks that traditional banks steer away from; they offer very small loans, yet still have such a high demand for these small bits of money; they empower women, who are often pushed aside in societies ruled by men; and they fund minor investments, which turn out major success stories. The excitement that microfinance generates has lead to a proliferation of similar lending programs throughout
the world, to the extent that the Microcredit Summit Campaign has recorded 2186 MFIs serving over 54.9 million clients (Daley-Harris, 2002). But the diversity of contracts in microfinance has made pinpointing the reason for its success increasingly difficult.

Credit markets for the poor suffer from the same informational asymmetries found in formal credit markets—lenders must ascertain a borrower’s creditworthiness as well as ensure repayment once a loan is disbursed. However, most microfinance institutions do not have the sophisticated credit scoring mechanisms, collateral requirements, and sound legal systems that allow banks to overcome these difficulties in developed markets. Innovative measures seen in microfinance help alleviate these problems.

Early interest in microfinance focused on the group lending methods used to solve the adverse selection and moral hazard problems. To lessen adverse selection, members are jointly liable for each others’ loans, compelling group members, who have better information than the lender, to choose individuals they believe most likely to repay. In addition, once groups are formed, each member has the incentive to monitor the others’ behavior, reducing both moral hazard and the lender’s monitoring costs. While these mechanisms are clearly present in the group lending programs, newer structures of microfinance, such as individual lending and village banks, cannot rely on these incentives, as the group is either non-existent or too large to have the necessary information to ensure repayment.

More recent research has focused on incentives present under both group and individual lending schemes. A significant amount of this research centers on the repeated interaction between borrower and lender, also known as ‘dynamic incentives.’ Although first attributed to Besley (1995), Morduch (1999) has been instrumental in cataloging the dynamic incentives at play in microfinance. When a borrower has continual credit needs, access to future loans can provide a strong reason to avoid default on a current loan. Furthermore, continual increases in loan size, or ‘progressive lending,’ improve a borrower’s incentives to repay over time (see Mosley, 1996; Jain and Mansuri, 2003).

Ensuring repayment incentives through refinancing is modeled in the context of microfinance by Hulme and Mosley (1996) and Armendáriz de Aghion and Morduch (2000). Using a two-period model, repayment of the first loan is induced with the promise of a second loan. However, in these two period models, the borrower always defaults on the second loan. In practice, MFIs stipulate that once a microentrepreneur defaults on a loan, she becomes forever ineligible for future loans. This tactic is almost universally employed by MFIs and is a fairly harsh means to ensure repayment.

Madajewicz (1997) notes that “these incentives are often quite extreme,” maybe too extreme, considering the dearth of alternative sources of credit for the poor in developing countries. Mosely, in a study of the Bolivian microfinance sector, tells the story of one microentrepreneur who lost his investment in a burglary. As a result of one member being burglarized, “the entire group fell into default, and remains so, banned from borrowing any more from BancoSol” (Mosley, 1999). A borrower with repayment problems faces a difficult choice: do I sell productive assets or withdraw my children from school in order to pay my loan, or do I surrender all access to future loans? Unfortunately, successful households may slide back into poverty in order to retain the possibility of loans in the future.

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1 For example, see Varian (1990), Stiglitz (1990), and Ghatak (1999).
Although commonly employed and considered successful, the non-refinancing threats may unnecessarily diminish borrower welfare. This model will question the need for such severe non-refinancing threats in microfinance, especially in light of countries with well-developed financial sectors, where default today does not mean a lifetime without credit. By endogenizing the default penalty, borrowers who fall upon bad luck may be able to obtain loans in the future, clearly improving outcomes for the poor.

Finally, academic models of microfinance base moral hazard on the choice of project to undertake but assume that a borrower will repay when she is able.2 Two deviations from this pattern are Besley and Coate (1995) and Armendáriz de Aghion (1999), who both address strategic default in group lending contracts. When successful group members are able to repay the portion of the loan for a defaulting group member, but refuse to do so, this can be construed as strategic default. Besley and Coate show how social collateral can reduce strategic default in some group lending situations. Furthermore, Armendáriz de Aghion finds that peer monitoring reduces strategic default when groups are exogenously formed. This paper will show how dynamic incentives, in the form of additional loans, can reduce strategic default without relying on the group incentives currently used in the literature.

When looking at typical loans given for microfinance projects, a model of strategic default may be more realistic than one of moral hazard in the choice of project. Microenterprise credit does not seek, in general, to allow individuals to start businesses, but to allow pre-existing businesses to grow by relaxing liquidity constraints. Some MFIs, such as the Lima-based Mibanco, have explicit rules that a microentrepreneur’s business must have been operating for 6 months before they can obtain a loan. Therefore, the choice that a borrower faces is not what project to undertake, but what to do with enterprise profits when the loan comes due.

The decision to repay the loan can then be modeled as a choice of whether or not to divert funds intended for loan repayments to other household wants or needs. This paper will distinguish between two types of default, strategic default and default due to a negative economic shock. One can easily construct either case: a family may choose to buy household luxury items instead of repaying a loan, or they may need to pay the funeral expenses of a family member. In light of the risks that poor entrepreneurs in developing countries face and the asymmetric information between borrower and lender regarding such risks, a model in which the lender provides incentives to discourage strategic default may be most appropriate.

There are the well known, systemic shocks, such as the flooding in Bangladesh, Hurricane Mitch, which struck microentrepreneurs in Honduras, or armed conflict, which can disrupt an entire economy. This paper will not address these issues as there is an existing literature among practitioners.3 However, there is an increasing amount of evidence that large systemic shocks are not the only things that put microentrepreneurs at risk.

2 See, for example, Stiglitz (1990), Ghatak (1999), or Madajewicz (1997). Madajewicz notes that this assumption may be justifiable based on access to future loans—which is modeled explicitly in this paper.

3 The literature on microfinance in post-disaster or post-conflict situations, written mainly by practitioners and donor agencies, advocates full recovery of those loans through debt rescheduling and new loans, but not through debt forgiveness. See, for example, Nagarajan (1998) and Doyle (1998).
risk. A study in Peru found that half of the sample experienced one or more shocks in the 2 years preceding the survey, the largest of which was burglary (Dunn, 1999). Without functioning insurance markets, theft, illness of a family member, or loss of another source of household income can quickly obliterate the ability to repay a loan on time and in full. As a result of these risks, they become ineligible for future loans, and, as the system currently functions, the “black mark” remains on their credit record indefinitely.

We propose an improvement over lending contracts currently used in practice, by including several features of microfinance not yet brought forth in the literature in addition to the moral hazard and adverse selection ever present in microfinance models. First, we can formulate a lending contract that continues for a potentially infinite amount of time and provides repayment incentives at all points in the game. Secondly, we can model the inherent riskiness of the income streams of the poor by including a negative income shock. Finally, the model will allow us to endogenize the amount of time that a borrower who defaults must remain without a loan. It will be shown that the necessary ‘punishment phase’ can be less than infinity, especially when an individual has much to gain from the lending relationship.

2. The model

The model begins with two categories of players, a single microfinance institution (“lender”) and a group of microentrepreneurs (“borrowers”). Borrowers have identical projects and borrowing needs, and differ only in the level of risk that they face. We will assume that the lender acts as a benevolent non-profit would, such that the lender aims to maximize the payoff of each borrower. Madajewicz (1997) structures her model similarly, in that the lender chooses to maximize the borrower’s utility, subject to the constraint that the lender breaks even. Following Conning (1999), this model maximizes expected microenterprise profits from borrowing.

The maximization of borrower profit is subject to three constraints: the borrower must be willing to accept a loan, must have the correct incentives to repay the loan when she is able, and the microfinance institution must maintain a sustainable lending operation over the entire loan portfolio by charging the appropriate interest rate. We define sustainability as covering both the cost of funds and administrative costs that the lender experiences. The lender and microentrepreneurs will maintain a financial relationship over several (and possibly an infinite number of) periods.

By adapting Green and Porter’s (1984) dynamic collusion game, we can model the repeated lender–borrower relationship found in microfinance. In each period, the borrower and lender may engage in a “lending phase,” where when one loan is successfully repaid, another loan is given. If, in any period, the borrower defaults, the borrower and lender then engage in a “punishment phase,” where no new loans are

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4 Note that this paper does not model the optimal contract but an improvement over current contracts that can be easily implemented.

5 Green and Porter use a two-phase game to model the dynamic interaction of two identical firms who collude under output uncertainty. This paper makes use of the two-phase structure, in the presence of default uncertainty.
extended to the borrower in default. After the punishment has been served, the borrower may return to the lending phase, with prior unpaid debts forgotten.\footnote{It might be possible for the lender to obtain partial repayment from a borrower either before default, or at a later date. It is conceivable that partial payment would lessen the amount of time that a borrower would need to spend in the punishment phase, clearly improving the borrower’s outcome.}

As non-refinancing threats require a lack of competition or shared access to borrower information to induce repayment (\cite{Ghosh and Ray, 1999}), we assume the presence of a single lender or sharing of black (default) credit information if multiple lenders are present. Otherwise, it is conceivable that a borrower could default with one lender and move on to borrow at a different MFI. As competition in microfinance is increasingly likely, there is a burgeoning interest in credit bureaus among MFIs. Information sharing is now part of microfinance ‘best practice’ (\cite{Campion and Valenzuela, 2001}), and in practice, credit bureaus created specifically for microfinance appear in every region of the globe (\cite{Isem, 2002}).

In this model there are two possible reasons for default—strategic default or default as a result of a negative economic shock. The lending contract provides incentives to discourage strategic default, but default due to an economic shock is unavoidable. The shock is unanticipated by the borrower and uncorrelated across borrowers (so does not represent systemic shocks), and is sufficiently large to automatically result in default. This would be the case if the borrower is uninsured, cannot self-insure (through savings or family assistance) and is therefore completely unable to deal with the shock.\footnote{The borrower may also be able to repay the loan out of savings or the income of another member of the household, diminishing the correlation of shock and default. This assumption of a one to one correspondence simplifies the model but would not change the results substantially, as shocks severe enough (or frequent enough) would still lead to default.} Furthermore, we assume that the lender is unable to observe whether default is due to a negative economic shock or strategic default by the borrower.

A final feature of the model is to include adverse selection by having borrowers differ in the level of risk that they face. Risk is modeled in this paper as \( x_i \), the probability for a borrower of type \( i \) of being hit by a negative and unanticipated economic shock that disturbs loan repayment. Borrowers have the same profitability, loan needs and discount factor, but can be either ‘Safe’ or ‘Risky.’ A portion of the population, \( \beta_0 \), faces lower risk compared to the portion \( (1 - \beta_0) \) who face higher risk, with \( \alpha_R > \alpha_S \). Individual borrowers know their type, but the lender only knows the distribution of types in the population.

Assuming that borrowers do not accrue savings, shocks are independent and identically distributed over time, and that the microfinance institution is well established\footnote{We exclude MFIs that are just beginning operations due to the significant amounts of learning (which would cause changing costs) and increases in the size of the lending portfolio. MFIs may also start off with higher levels of default in order to establish credibility of punishment and survival.}, the equilibrium is stationary. We will solve the lending game by determining the sustainable interest rate, \( r \), and the optimal length of the punishment phase. The punishment phase lasts for \( T \) periods, where \( T \) should be sufficiently long to prevent a borrower from strategic default, but not so long as not to unduly punish the borrower that experiences a negative economic shock. The rest of this section will lay out the payoffs for both the borrower and lender under both the lending and punishment phases. The following...
sections will solve for the interest rate and length of the punishment phase, and discuss the implications of the findings.

2.1. The borrower’s payoff from accepting a loan

In the beginning of any period in the lending phase, a borrower may take a loan of \( B \) and agrees to repay the lender \((1 + r)B\) at the end of the period, where \( r \) is the interest rate on the loan.\(^9\) We assume a fairly simple production function that would apply in microenterprises such as retail or food service, where more up front capital allows for a larger inventory. Given the loan of \( B \), a microentrepreneur is able to earn \( wB \) from her business as a result of investment made from the loan.\(^10\)

The borrower may, during each lending period, suffer from a shock that disturbs her ability to repay the loan. For a borrower of type \( i \), there is a \( (1-\alpha_i) \) chance that the borrower earns \( wB \), repays the loan, and receives another loan in the following period, and an \( \alpha_i \) chance that she will experience a negative shock and default, entering the punishment phase in the following period. In either case, the borrower discounts the next period’s expected payoff by \( \delta \).

Let us define two functions for a borrower of type \( i \), \( V_i^+ \), the payoff at the beginning of a period in the lending phase, and \( V_i^- \), the payoff at the beginning of the punishment phase. For a borrower in the lending phase, there is a \( (1-\alpha_i) \) chance of successfully repaying the loan, earning a profit, and beginning the lending phase again in the next period. There remains an \( \alpha_i \) chance of default, with certain entrance into the punishment phase in the next period. There remains an \( \alpha_i \) chance of default, with certain entrance into the punishment phase in the next period, leading to a function of

\[
V_i^+ = (1-\alpha_i)\left[(w-(1+r))B + \delta V_i^+\right] + \alpha_i\delta V_i^-,
\]

as the borrower’s payoff under the lending phase.

A borrower in the punishment phase receives no loans and therefore earns nothing from the borrowing relationship for \( T \) periods.\(^11\) After the punishment time has been served, the microentrepreneur may return to the lending phase in the \( T+1 \)st period. Depending on the number of eligible borrowers seeking a loan, a borrower may be credit constrained if the lender limits the size of the portfolio (which we impose in Section 2.3). To allow for this possibility, we define \( \gamma \), the probability that a borrower will be able to return to the lending phase in any period after the \( T \)th period. If a borrower cannot return to the lending phase in the \( T+1 \)st period, the probability of

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\(^9\) The aim of this section is to establish payoffs. The moral hazard problem will be addressed in the next section through the borrower’s incentive constraint.

\(^10\) The production function can be generalized without drastically changing the outcomes of the model. See footnote 14 in Section 3.2 for more detailed information.

\(^11\) This can be relaxed somewhat by allowing the borrower to make some money in the punishment phase (based on, for example, earlier investments or an existing stock of capital), although the per-period punishment phase payoff must necessarily be less than returns from borrowing. Such a change will most likely force \( T \) to be somewhat higher, as it weakens the deprivation of the punishment phase.
being able to borrow in the $T+2$nd period is $\gamma(1 - \gamma)$, which must then be discounted. Together, this gives the payoff under the punishment phase as

$$V_i^- = \sum_{t=0}^{\infty} \delta'(1 - \gamma)^{t} \frac{\gamma \delta T}{1 - \delta(1 - \gamma)} V_i^+.$$  \hfill (2)

Combining Eqs. (1) and (2) will give the payoff functions

$$V_i^+ = \frac{(1 - \alpha_i)(w - (1 + r))B}{1 - \delta(1 - \alpha_i) - \frac{\alpha_i \gamma \delta T + 1}{1 - \delta(1 - \gamma)}},$$

and

$$V_i^- = \frac{\gamma \delta T (1 - \alpha_i)(w - (1 + r))B}{[1 - \delta(1 - \gamma)] [1 - \delta(1 - \alpha_i)] - \alpha_i \gamma \delta T + 1},$$

in terms of our two choice variables, $r$ and $T$, and the exogenous parameters $\alpha_i$, $\gamma$, $w$, $B$ and $\delta$.

2.2. The borrower’s constraints

The borrower has two constraints for an equilibrium to exist: participation and incentive constraints. For a borrower to be willing to accept the loan, the returns from borrowing must be greater than the cost of borrowing in the no-shock state. Thus, the borrower’s participation constraint, $w > 1 + r$, implies that the microentrepreneur will only take the loan when they are able to make a profit on the venture. This is the same for both high and low risk borrowers, since repayment is only possible in the no-shock state.

The only incentive mechanism that the lender has in this model is the threat of not refinancing, as collateral is not widely available among the poor (and therefore uncommon in microfinance). In order to induce the borrower to pay when she is able, the deprivation imposed by the punishment phase must be larger than the gain from non-payment. Strategic default will not occur if the payoff from investing, repaying the loan, and entering the lending phase in the next period is greater than the highest one period gain from cheating and going into the punishment phase with certainty in the next period, or,

$$[w - (1 + r)]B + \delta V_i^+ \geq wB + \delta V_i^-.$$  \hfill (5)

The best that a potential defaulter can do is to invest the money, experience no shocks, and keep the earnings ($wB$) without paying the microfinance institution any of the principal or interest.

By rearranging Eq. (5), we obtain the following incentive constraint,

$$\delta [V_i^+ - V_i^-] \geq (1 + r)B,$$

which implies that the loss in future earnings from purposefully defaulting on a loan must be greater than the one period gain from defaulting, in order to give the incentives to repay the loan when the microentrepreneur is able.
Substituting Eqs. (3) and (4) into Eq. (6) yields the incentive constraint for a borrower of type \(i\), in terms of \(r\), \(T\), and constants (see Appendix for derivations),

\[
\frac{\delta(1 - \gamma) [1 - \gamma(1 - \gamma) - \gamma \delta^T]}{1 - \delta(1 - \gamma) - \gamma \delta^{T+1}} \geq \frac{(1 + r)}{w}.
\]  

(7)

This is a restriction on the relationship of the length of the punishment phase to the interest rate, given the parameters of the model. This relationship will be used in later, in conjunction with the lender’s problem, in order to determine when it is possible to attain a borrowing equilibrium of the sort described in this paper.

2.3. The microfinance institution’s problem

The lender also experiences different payoffs based on the stage of the game (lending or punishment phases), although the lender will operate in both stages simultaneously—the lender will continue lending to the portion of the portfolio that continues to repay, and withhold lending from those who have defaulted. We normalize the number of borrowers in the lending phase to one and replace defaulting borrowers from the punishment phase, which has the same distribution of types as the population as a whole. The portfolio converges to a stable point where there are \(\beta\) safe types in the portfolio and \((1 - \beta)\) risky types, where

\[
\beta = \frac{\alpha_R \beta_0}{\alpha_S (1 - \beta_0) + \alpha_R \beta_0}.
\]  

(8)

The proportion of safe types in the portfolio is greater than that of the general population \((\beta > \beta_0)\) as safe types default less often than risky types. The average risk of the portfolio is also stable at \(\hat{\alpha} = \beta \alpha_S + (1 - \beta) \alpha_R\), implying that in any given period, a fraction \(\alpha\) of the borrowers are hit by a shock and will be sent to the punishment phase of the game, while the remainder, \((1 - \hat{\alpha})\), who are not affected will remain in the lending phase.

We can define the lender’s value functions in the lending phase, \(\Pi^+\), and in the punishment phase \(\Pi^-\) in terms of constants, the interest rate and the length of the punishment phase, or

\[
\Pi^+ = \frac{[(1 + r)(1 - \hat{\alpha}) - (1 + z)]B}{1 - \delta(1 - \hat{\alpha}) - \frac{\hat{\alpha} \gamma \delta^{T+1}}{1 - \delta(1 - \gamma)}},
\]  

(9)

and

\[
\Pi^- = \frac{\gamma \delta^T}{1 - \delta(1 - \gamma)} \frac{\Pi^+}{[1 - \delta(1 - \gamma)][1 - \delta(1 - \hat{\alpha})] - \hat{\alpha} \gamma \delta^{T+1}}.
\]  

(10)

where \(z\) is the lending cost per dollar lent, including operating costs.

12 A full explanation of the MFIs problem and the derivation of Eqs. (8), (9) and (10) can be found in the Appendix.
As the MFI aims to maximize borrower profits from taking a loan, it will not be a profit-maximizing lender, but only required to have sustainable lending operations. Thus, the lender’s sustainability constraint will be that on average, the MFI cannot earn negative profit, or that (9) be non-negative.

3. Solving the model

To find the optimal length of the punishment phase, we would maximize the borrower’s payoff (3) choosing \( r \) and \( T \), subject to the incentive constraint (7) and the lender’s sustainability constraint. Then, the borrower’s participation constraint must be checked to ensure that a sustainable borrowing equilibrium exists. We can most easily find the answer by noting the properties of the borrower’s value function and incentive constraint and the lender’s sustainability constraint.

3.1. Solving for the optimal interest rate

First, it is straightforward to see how the interest rate should be set as low as possible in order to maximize the payoff to the borrower, but a zero or negative interest rate will not satisfy the sustainability constraint of the lender. Therefore the interest rate must be positive, but only so large as to allow the lender to break even, or \( \Pi^+ = 0 \). This holds when \((1+r)(1-\hat{z})-(1+z)=0\), which implies an interest rate of

\[
\hat{r} = \frac{\hat{z} + z}{1 - \hat{z}}. \tag{11}
\]

The interest rate that both maximizes borrower welfare and makes lending sustainable will exactly cover the total cost of lending, adjusted for the portion of the portfolio that is non-performing.

3.2. Solving for the length of the punishment phase\(^{13}\)

It is also straightforward to show that a borrower would prefer to have no break in lending operations when she defaults. However, for most borrowers, a non-trivial punishment phase will be necessary to preserve repayment incentives. In equilibrium, we would like \( T \) to be small–to maximize the payoff of borrowing for the microentrepreneur–but \( T \) must be large enough to satisfy the incentive constraints for both borrower types. Solving for the \( T^* \) which leads to a binding incentive constraint for the risky borrower (the one with the weaker repayment incentives) will give the following expression,

\[
T^* = \frac{\ln \left( \frac{[1-\delta(1-\gamma)][1+z-w\delta(1-\alpha_R)(1-\hat{z})]}{\gamma[1+z-w(1-\alpha_R)(1-\hat{z})]} \right)}{\ln \delta} - 1, \quad (12)
\]

\(^{13}\) Derivations for this section can be found in the Appendix.
which is a sufficient length of time in the punishment phase to keep repayment incentives intact.\footnote{As stated in an earlier footnote, the borrower’s production can be generalized, such that the length of the punishment phase can be made to depend on the amount borrowed. If we change from the linear production function, \( wB \), to a more general function, \( f(B) \), larger loan sizes can result in either higher or lower punishment depending on the functional form.}

Note that over several loan cycles, risky borrowers effectively spend more time in the punishment phase than would a safe borrower, since they will default more often. This is similar to the finding by Ghatak (1999) that the effective cost of borrowing may be lower for safe types, even with a single group lending contract. Finally, the time spent is not optimal, but simply sufficient for this model. Clearly, both types could be made better off by using borrower histories, and Sadoulet (2004) presents a different model with such characteristics.

For \( T^* \) to exist and be positive—and therefore a true ‘punishment’—a few conditions must be met. For existence, we require that the lowest expected benefit of taking a loan is greater than its cost, \( w(1 - \alpha_R) > 1 + r^* \), and that the borrowers must sufficiently value the future (\( \delta \) large enough), a common restriction in game-theoretic models. For \( T^* > 0 \), there is a final restriction that \( \gamma \), the probability of return to the lending phase, is large enough. As \( \gamma \) decreases, a borrower has a lower chance of being able to obtain a loan after having served their time in the punishment phase, which effectively increases their punishment. In this case, there is a rift in the required \( T^* \) versus the actual amount of time a person is without a loan after default, and \( T^* = 0 \) may still satisfy the incentive constraint. The required magnitudes of the parameters will be examined further in Section 4.2.

Note that the length of the punishment phase inherently depends on the interest rate, as it was a prominent factor in the borrower’s incentive constraint (7). If the interest rate were to rise, the benefits of strategic default also rise. This will necessitate a longer length of time without loans in order to sustain a borrowing equilibrium. On the other hand, if the interest rate falls, benefits of strategic default are lower and the length of the punishment phase will decrease.

### 3.3. Existence of the equilibrium

The final key to the model is ensuring that the participation constraint is satisfied. The model starts under the premise that the borrower would want to take out the loan, or that \( w > (1 + r) \), which is the same for both types of borrowers. However, one condition required for \( T \) to exist, \( w(1 - \alpha_R) > 1 + r^* \), dominates the participation constraint, and is thus now required for existence of the equilibrium.

By rearranging terms, the condition for existence shows four things. For a sustainable individual borrowing equilibrium using only dynamic incentives, (13) implies that the borrower’s profitability must be sufficiently high, the lending costs must be kept low, the risky group of borrowers cannot be too risky, and the average probability of default must be sufficiently low,

\[
(1 - \alpha_R)w > \frac{1 + \bar{z}}{(1 - \hat{x})}.
\]  

(13)
While any one of these findings, on their own, are fairly straightforward, they shed some light on current debates among microfinance practitioners. 

Elisabeth Rhyne states that “everyone wants to reach the poor and everyone believes sustainability is important. This is not an either-or debate” (quoted in Wright and Dondo, 2001). However, Woller et al. (1999) argue that the definition of “poor” is debatable across those in the field, and that subsidies may be required in some cases. Microfinance institutions which operate in risky environments or are attempting to lend in more remote areas may have to make a decision to either curtail outreach to these clients or face the fact that full financial self-sufficiency may not be possible, as noted by Morduch (2000). Furthermore, targeting the poor is not the main goal of all microfinance institutions. While some may refer to this as “mission drift,” Robert Peck Christen (2000) explains that the first MFIs in Latin America concentrated more on developing enterprises and expanding employment as opposed to providing financial services to the poorest of the poor. 

This might explain how programs that do not have a specific focus on poverty alleviation can profitably lend to individuals—they do not aim to start new, potentially low-profit businesses, but to expand successful ones where \( w \) would be high. Realize that this model imposes sustainability. Institutions who find themselves with less profitable or more risky borrowers may face one of three choices, none of which are palatable to most donor agencies or practitioners. First, The MFI may choose to become (or remain) unsustainable, but microfinance best practice discourages the continual subsidization of MFIs. Second, the MFI may choose to target borrowers with more profitable microenterprises, although most microenterprise credit programs aim to reach the poorest segments of society, whose businesses are generally smaller and less profitable. The final option is to completely shut down lending operations or curtail outreach to certain segments of society, which will clearly not maximize borrower payoffs. Borrower profitability and risk—and their relationship to each other—clearly have crucial implications for the current “best practice” of becoming financially self sufficient.

4. Implications for microfinance institutions

4.1. Effects on sustainable lending

Lending costs, borrower risk, and profitability are so closely connected that any movement in one of these factors will require a corresponding change in the interest rate and length of punishment. Such movements might also cause sustainability (or the existence of borrowing) to vanish. As donors withdraw funding, MFIs may find it increasingly difficult to sustainably continue lending operations. The area below the lines in Fig. 1 shows the \((\hat{z}, w)\) combinations that allow a borrowing equilibrium with a \(T\) period punishment to exist. As shown, higher lending costs will cause the set of feasible

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15 There is some dissention, as seen in Dunford (2000), although most government donor agencies and international institutions support subsidization only for institutional development, instead of operating costs.

16 For Figs. 1 and 2, assume that \(\delta=0.8, \gamma=0.5\) and that there are equal numbers of risky and safe borrowers \((\beta=0.5)\), with \(z_R=1.5\hat{z}\).
solutions to shrink, implying that a sustainable borrowing equilibrium is less likely, or will require MFIs to move to more profitable or less risky clientele.

Furthermore, as shown in Fig. 2, decreases in profitability also erode the possibility of an equilibrium. Expansion into remote areas, or where entrepreneurs may be less profitable, will prove difficult. As cost-cutting will be a challenge when attempting to
expand and finding safer clients in remote areas may prove difficult, the only solution may be to curtail outreach.

4.2. Length of non-refinancing threats

This model suggests that it is not necessary to permanently reject borrowers who default, in the face of negative economic shocks that may diminish the borrower’s ability to repay the loan. It is only necessary to have a sufficiently long period in which lending operations cease, and thus the optimal length of the punishment phase may be less than infinity for certain borrowers and even zero under some special cases.

As seen in Fig. 3, for high levels of risk, a sustainable borrowing equilibrium predicated on a $T$ period punishment phase does not exist. However, provided that risk is low enough, an equilibrium can be sustained. The surprising part of the analysis is that for low values of gamma—or low probability of return to the lending phase—$T^*$ can be zero and still satisfy the incentive constraint. $T^*=0$ does not mean that defaulters will remain in the lending phase. They still enter the punishment phase, are immediately eligible to return to the lending phase, but have a low probability of reentry. This, in effect, means that the borrower is still punished enough to discourage strategic default. Granted, this is a special case that requires low values of $\gamma$.

More often than not, $T^*$ is strictly positive. Consider the examples in Tables 1 and 2, which show a plausible range of possible parameter values. Each table deals with a different risk group, with Table 1 showing a fairly low-risk group of borrowers, where average default is only 5%, and Table 2 showing a more risky group of borrowers with $\hat{\gamma}=0.15$. Panels within each table show a range of lending cost, from microfinance institutions with subsidized interest rates or lower administrative costs to those who must borrow on capital markets. In every comparison, three levels of the
Table 1
Length of the punishment phase, lower average risk

\[ \hat{z} = 0.05 \]

<table>
<thead>
<tr>
<th>( z = 0.05 )</th>
<th>( z = 0.10 )</th>
<th>( z = 0.18 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>( \hat{\delta} )</td>
<td>( w )</td>
</tr>
<tr>
<td>0.8</td>
<td>0.68</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>2.5</td>
<td>0.35</td>
<td>0.74</td>
</tr>
<tr>
<td>2</td>
<td>1.26</td>
<td>2.39</td>
</tr>
</tbody>
</table>

\(^a\) Figures are based on equal numbers of risky and safe borrowers (\( \beta = 0.5 \), \( \gamma = 0.5 \), and \( \alpha_R = 1.5 \hat{z} \)).

\(^b\) dne means that an individual borrowing equilibrium with these parameters does not exist.

microentrepreneur’s profitability and discount factor are shown. The values for \( w \) of 2, 2.5, and 3 were chosen with vendors in mind, who can double or triple the loan amount through sales.\(^17\) The values for the discount factor came from Frederick et al. (2002).\(^18\)

Even though these represent a limited number of examples, several points are clear. First, there are some parameter combinations for which a value of \( T \) does not exist, meaning that an individual lending equilibrium with a finite punishment may not always be possible. Additional incentives may be necessary to discourage strategic default, and we do see collateral, required savings, and sometimes co-signers in addition to the dynamic incentives used in individual lending.

In the examples where \( T \) does exist, we see a wide range of possible outcomes. First, there is only one instance where \( T^* = 0 \) will satisfy the incentive constraint (7), which is for a highly profitable borrower who greatly values the future. In this instance, the value of the loan is too high for the borrower to ever strategically default. More often though, a non-zero punishment phase is necessary. Recall that in this model there was a lending “period” and \( T \) was the amount of lending periods a defaulter needed to be without loans to discourage strategic default. For a microfinance institution which offers one year loan terms, we might optimally see a defaulter be without loans anywhere from 11 weeks to just over 17 years.\(^19\) This is certainly less than the current practice of arbitrarily making \( T \) equal to infinity, especially when many values require only one or two lending periods as punishment. However, 17 years may seem like infinity to a poor borrower with no other financing options.

\(^{17}\) Appendix 6 in Edgecomb and Garber (1998) shows several borrowers with profitability in this range.

\(^{18}\) Table 1 from Frederick et al. (2002) displayed estimated discount factors from 42 papers published over the period 1978–2002. Most papers gave a range of possible \( \hat{\delta} \) values and Frederick et al reported the maximum and minimum values estimated in each of the papers. The three values for delta that are used in this example are drawn from the average high and low values among the included studies (0.8 and 0.55), as well as the mean of all discount factors estimated in those same studies (0.68).

\(^{19}\) Eleven weeks corresponds to the lowest value for \( T \), 0.20 periods, over a 52-week loan cycle, which would imply a punishment phase of 10.4 weeks.
Table 2
Length of the punishment phase, higher average risk

<table>
<thead>
<tr>
<th>$\hat{z}$=0.15</th>
<th>$z$=0.05</th>
<th>$z$=0.10</th>
<th>$z$=0.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>$\delta$</td>
<td>w</td>
<td>$\delta$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.68</td>
<td>0.55</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>0.68</td>
<td>1.26</td>
<td>3.77</td>
</tr>
<tr>
<td>2.5</td>
<td>1.78</td>
<td>3.84</td>
<td>dne</td>
</tr>
<tr>
<td>2</td>
<td>17.02</td>
<td>dne</td>
<td>dne</td>
</tr>
</tbody>
</table>

$^a$ Figures are based on equal numbers of risky and safe borrowers ($\beta=0.5$, $\gamma=0.5$, and $z_R=1.5\hat{z}$).
$^b$ dne means that an individual borrowing equilibrium with these parameters does not exist.

4.3. Distortion created by imperfect information

Even with a long lending history, MFIs are still unable to discern the type of a particular borrower in this model. A safe borrower with a stream of bad luck could be mistaken for risky, and a risky borrower with a string of good luck could be taken for a safe borrower. If the lender could perfectly discern a safe borrower from a risky borrower, the payoff to safe borrowers would rise at the expense of the risky borrowers.

Assume that each borrower’s type is known to the lender. Safe borrowers would be charged an interest rate based only on $\alpha_S$, and risky borrowers face an interest rate based on $\alpha_R$, with a borrower of type $i$ facing an interest rate

$$r_i^* = \frac{\alpha_i + z}{1 - \alpha_i}.$$  

Given that $\alpha_R > \hat{z} > \alpha_S$, the safe borrowers are charged a lower interest rate under perfect information, while risky borrowers are charged a higher interest rate under the same conditions.

If interest rates differ under perfect information, it can be safely said that $T^*$ will also change. Again, the length of the punishment phase is now distinct and based solely on the borrowers’ own type,$^{20}$ with

$$T_i^* = \frac{\ln \left( \frac{1 - \delta(1 - \gamma)}{\gamma} \frac{1 + z - w(1 - \alpha_i)^2}{(1 - \alpha_i)^2} \right)}{\ln \delta} - 1.$$  

It can be shown that the perfect information solution for the safe types, $T_S^*$, is shorter than the length of the punishment phase under imperfect information, $T^*$. Similarly, the risky borrowers must spend longer in the punishment phase under perfect information than they would under imperfect information, or $T_R^* > T^*$.

$^{20}$ $T^*$ was derived in the Appendix $T_i^*$ differs only in the interest rate and the level of risk used.
With both a lower interest rate and a shorter punishment phase, safe borrowers have higher payoffs from borrowing in the perfect information case. The risky borrower faces a higher interest rate and longer punishment, implying that they are better off with imperfect information. In either case, however, the safe borrower always has a higher payoff from a loan than the risky borrower does.

5. Conclusions

By acknowledging the unexpected events that microentrepreneurs face, we can devise a model that describes a common reason for default and begin to see how better functioning financial and insurance markets could provide relief to the poor in developing countries. Using dynamic incentives, in the context of individual lending, we show how microfinance can become a sustainable development alternative, especially in areas where markets are thicker and microentrepreneurs are very profitable, in more urban locations where costs of outreach are lower, and in less risky environments.

The results lend support to MFI attempts to improve and expand upon savings and insurance products. Any attempt to decrease vulnerability, by helping borrowers to improve risk mitigation strategies and risk coping strategies, would allow for more outreach. This has been a priority among practitioners (see Sebsted and Cohen, 2000), but is only recently gaining the interest of academics. If borrowers could be sheltered from the unplanned expenditures or loss of income, or be better able to cope with economic loss when it occurs, their ability to repay would be higher, and thus default would be lower. As the default rate falls, the interest rate that microfinance organizations must charge to remain sustainable will also fall, allowing more borrowers to have access to credit.

However, as the regulations in some countries do not allow microfinance institutions to also offer a savings product, legislation to aid institutions is desperately needed. Furthermore, the failures in insurance markets—including unemployment insurance, life insurance and insurance against theft—must be studied in greater detail. Sadoulet (2004) derives a credit-with-insurance contract for microfinance, but more investigation of the microfinance-insurance connection will most certainly improve borrower outcomes.

Furthermore, by cutting interest subsidies to microfinance institutions that cater to the least profitable microenterprises and rural businesses, or operate in areas most prone to adverse economic shocks, donor agencies may be forcing MFIIs out of business. Sustainability is not possible under all circumstances, and we—both academics and practitioners—need to sort out when a microfinance institution can be sustainable, and when subsidies may be warranted.

Acknowledgements

The author would like to thank seminar participants at NEUDC and the University of Maryland, as well as Roger Betancourt and two anonymous reviewers for their thoughtful suggestions. Any remaining errors are, of course, my own.
Appendix A. Derivation of Eq. (7)

Substitute (3) and (4) into (6) and rearrange terms

\[
\delta \left[ 1 - \frac{\gamma^{\delta^T}}{1 - \delta(1 - \gamma)} \right] \frac{(1 - \alpha_i)[w - (1 + r)]B}{1 - (1 - \alpha_i)\delta - \frac{\alpha_i \gamma^{\delta^T+1}}{1 - \delta(1 - \gamma)}} \geq (1 + r)B \\
\]

\[
\Rightarrow \delta \left[ 1 - \delta(1 - \gamma) - \gamma^{\delta^T} \right] (1 - \alpha_i)[w - (1 + r)] \\
\geq (1 + r) \left[ \frac{1 - \delta(1 - \gamma) - [(1 - \alpha_i)\delta][1 - \delta(1 - \gamma)] - \alpha_i \gamma^{\delta^T+1}}{1 - \delta(1 - \gamma)} \right] \\
\Rightarrow \delta \left[ 1 - \delta(1 - \gamma) - \gamma^{\delta^T} \right] (1 - \alpha_i)w \geq (1 + r) \left[ 1 - \delta(1 - \gamma) - \gamma^{\delta^T+1} \right] \\
\Rightarrow \frac{\delta(1 - \alpha_i)[1 - \delta(1 - \gamma) - \gamma^{\delta^T}]}{1 - \delta(1 - \gamma) - \gamma^{\delta^T+1}} \geq \frac{(1 + r)}{w} \quad (7) \]

Note that the sign does not change since the denominator of (3) is positive.

\[
1 - (1 - \alpha_i)\delta - \frac{\alpha_i \gamma^{\delta^T+1}}{1 - \delta(1 - \gamma)} > 0 \Leftrightarrow (1 - \delta) + \alpha_i \delta \left( 1 - \frac{\gamma^{\delta^T}}{1 - \delta(1 - \gamma)} \right) > 0 \\
\Leftrightarrow (1 - \delta) + \alpha_i \delta \left( 1 - \delta(1 - \gamma) - \gamma^{\delta^T} \right) > 0 \quad \text{(A.1)}
\]

For (A.1) to hold, \( 1 - \delta(1 - \gamma) - \gamma^{\delta^T} > 0 \). This can be rewritten as \( 1 - \delta(1 - \gamma)(1 - \delta^{T-1}) \), which is positive if \( T \geq 1 \), or could also be written as \( (1 - \delta) + \gamma \delta(1 - \delta^{T-1}) \) which is positive if \( T < 1 \).

Appendix B. Derivation of the Microfinance Institution’s Problem

We normalize the number of borrowers in the lending phase to one, and replace defaulters from the punishment phase, whose members mimic the distribution of the population as a whole. The proportion of safe types in the lender’s portfolio will be greater than that of the general population because safe types default less frequently than the risky types and a relatively larger number of safe types enter than had left. Defining \( \beta_t \) as the proportion of safe types in the portfolio in period \( t \), it evolves according to the following equation

\[
\beta_t = \beta_{t-1}(1 - \alpha_S) + \beta_0(\beta_{t-1}\alpha_S + (1 - \beta_{t-1})(1 - \alpha_R)).
\]
There is a point where the proportion of safe types in the portfolio converge, or \( \beta_t = \beta_{t-1} \), which we call \( \beta \), where

\[
\beta = \frac{\alpha_R \beta_0}{\alpha_S (1 - \beta_0) + \alpha_R \beta_0}.
\]  

(8)

In any period, the portfolio will consist of the borrowers of both types who successfully repaid their loans in the previous period \( (\beta (1 - \alpha_S) + (1 - \beta)(1 - \alpha_R)) \), and borrowers that replace those who defaulted \( (\beta \alpha_S + (1 - \beta)\alpha_R) \) which consists of \( \beta_0 \) safe types and \( (1 - \beta_0) \) risky types. The number defaulting in the next period, or the average portfolio risk, is equal to

\[
\alpha_S (1 - \alpha_S) + \alpha_R (1 - \alpha_R)(1 - \beta) + (\beta \alpha_S + (1 - \beta)\alpha_R)(\beta_0 \alpha_S + (1 - \beta_0)\alpha_R),
\]

where the first two terms represent the risk of the borrowers remaining in the lending phase and the final term represents the average risk of the borrowers returning from the punishment phase. The expression collapses to \( \hat{\alpha} = \beta \alpha_S + (1 - \beta)\alpha_R \) when we substitute in for \( \beta \) from (8) and rearrange terms. Thus, in any given period, a fraction \( \alpha \) of the borrowers are hit by a shock and will be sent to the punishment phase of the game, while the remainder, \( (1 - \alpha) \), who are not affected will remain in the lending phase.

Now that portfolio risk and distribution are stable, we can look at the lender’s problem in any period. It costs the lender \( (1 + z)B \) to lend to a borrower who takes a loan of size \( B \), where \( z \) is the lending costs per dollar lent. In return, the lender receives \( (1 + r)B \) from a borrower who repays the loan or nothing from a borrower who defaults. Therefore, the lender’s net gain in any period is \( [(1 + r)(1 - \hat{\alpha}) - (1 + z)]B \) plus whatever the lender expects to get in the subsequent periods. In the next period, discounted by \( \delta \), the MFI engages in the lending phase with the borrowers who have repaid in the previous period and enough returning borrowers to make the size of the portfolio constant, and engages in the punishment phase with those who have defaulted. We can thus define \( \Pi^+ \) as the lender’s payoff under the lending phase, or

\[
\Pi^+ = [(1 + r)(1 - \hat{\alpha}) - (1 + z)]B + \delta \left[ (1 - \hat{\alpha}) \Pi^+ + \hat{\alpha} \Pi^- \right],
\]  

(A.2)

where \( \Pi^- \) is the lender’s payoff for a borrower in the punishment phase.

During the punishment phase, the lender makes no loans to the defaulting borrower and thus earns nothing from the relationship for \( T \) periods. We also assume that there are no costs involved with a borrower in the punishment phase. In the \( T+1 \)st period, the lender resumes the lending phase with the newly eligible borrower with probability \( \gamma \), and we can define the payoff for the lender at the beginning of the punishment phase as

\[
\Pi^- = \frac{\gamma \delta^T}{1 - \delta (1 - \gamma)} \Pi^+.
\]  

(A.3)
By combining Eqs. (A.2) and (A.3) we can define the lender’s value functions in the lending phase, \( II^+ \), and in the punishment phase \( II^- \), in terms of constants, the interest rate and the length of the punishment phase, or

\[
II^+ = \frac{[(1 + r)(1 - \hat{x}) - (1 + z)]B}{1 - \delta(1 - \hat{x}) - \frac{\hat{x}\delta^{T+1}}{1 - \delta(1 - \gamma)}},
\]

and

\[
II^- = \frac{\gamma\delta^T[(1 + r)(1 - \hat{x}) - (1 + z)]B}{[1 - \delta(1 - \gamma)][1 - \delta(1 - \hat{x})] - \hat{x}\delta^{T+1}}.
\]

### Appendix C. Derivation of Eq. (12)

First, note that \( V^+ \) is a decreasing function of \( T \), or that the borrower would prefer to have no break in lending operations when they default,

\[
\frac{\partial V^+_i}{\partial T} = -\frac{(1 - \alpha_i)[w - (1 + r)]B}{1 - (1 - \alpha_i)\delta - \frac{\alpha_i\gamma\delta^{T+1}}{1 - \delta(1 - \gamma)}} < 0
\]

However, setting \( T = 0 \) to maximize borrower payoffs will generally not be incentive compatible.

Next, note that the left-hand side of the borrower’s incentive constraint (7) is an increasing function of \( T \),

\[
\frac{\partial}{\partial T} \left( \frac{\delta(1 - \alpha_i)(1 - \delta(1 - \gamma) - \gamma\delta^T)}{1 - \delta(1 - \gamma) - \gamma\delta^{T+1}} \right)
\]

\[
= \frac{[1 - \delta(1 - \gamma) - \gamma\delta^{T+1}) - \delta(1 - \delta(1 - \gamma) - \gamma\delta^T)](-\gamma(1 - \alpha_i)\delta^{T+1}\ln \delta)}{[1 - \delta(1 - \gamma) - \gamma\delta^{T+1}]^2}
\]

Thus the \( T \) that will solve the maximization problem will be \( T^* \) for which the incentive constraint binds, or

\[
\frac{\delta(1 - \alpha_i)(1 - \delta(1 - \gamma) - \gamma\delta^{T^*})}{1 - \delta(1 - \gamma) - \gamma\delta^{T^*+1}} = \frac{(1 + r)}{w}.
\]
Solving for $T^*$,

$$w\delta(1 - x_i)(1 - \delta(1 - \gamma) - \gamma \delta T^*) = (1 + r)(1 - \delta(1 - \gamma) - \gamma \delta T^* + 1)$$

$\Leftrightarrow \gamma[(1 + r) - w(1 - x_i)]\delta T^* + 1 = [(1 + r) - w \delta(1 - x_i)](1 - \delta(1 - \gamma))$

$\Leftrightarrow \delta T^* + 1 = \frac{[(1 + r) - w(1 - x_i)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_i)]}$

$\Leftrightarrow (T^* + 1)\ln \delta = \ln \left( \frac{[(1 + r) - w \delta(1 - x_i)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_i)]} \right)$

$\Leftrightarrow T^* = \frac{\ln \left( \frac{[(1 + r) - w \delta(1 - x_i)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_i)]} \right)}{\ln \delta} - 1.$

This would lead to two distinct lengths of the punishment phase, depending on whether $x_S$ or $x_R$ were used. However, as the lender cannot distinguish between the two types, a single $T^*$ will be employed. In order for both types of borrowers to have the proper repayment incentives, the larger $T^*$ must be used. As $T^*_S < T^*_R$ when $x_S < x_R$, then $T^*_R$ is employed.

**Proof.**

\[
T^*_S < T^*_R \Leftrightarrow \frac{\ln \left( \frac{[(1 + r) - w \delta(1 - x_S)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_S)]} \right)}{\ln \delta} - 1 < 0
\]

\[
\Leftrightarrow \ln \left( \frac{[(1 + r) - w \delta(1 - x_S)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_S)]} \right) > \ln \left( \frac{[(1 + r) - w \delta(1 - x_R)](1 - \delta(1 - \gamma))}{\gamma[(1 + r) - w(1 - x_R)]} \right)
\]

\[
\Leftrightarrow \delta(1 - x_S) + (1 - x_R) < (1 - x_S) + \delta(1 - x_R) < x_S(1 - \delta) < x_R(1 - \delta) \Rightarrow T^*_S < T^*_R.
\]

Finally, substituting in for $r^*$, we get $T^*$ in terms of parameters,

\[
T^* = \frac{\ln \left( \frac{[1 + z - w \delta(1 - x_R)(1 - \hat{\alpha})](1 - \delta(1 - \gamma))}{\gamma[1 + z - w(1 - x_R)(1 - \hat{\alpha})]} \right)}{\ln \delta} - 1.
\]
Appendix D. Requirements for a Plausible $T^*$

For $T^*$ to be a plausible length of the punishment phase, it must exist and should be positive. To ensure that $T^*$ exists, the natural logs of the equation must exist. The denominator of the first term, $\ln \delta$, certainly exists, as $0 < \delta < 1$. Therefore, we need to ensure that the numerator also exists, or that the argument of the numerator of the first term in (12) must be positive, or

$$\frac{1 + z - w\delta(1 - x_R)(1 - \hat{x})}{1 + z - w(1 - x_R)(1 - \hat{x})} > 0.$$  \hspace{1cm} (A.4)

First assume that $1 + z - w(1 - x_R)(1 - \hat{x}) < 0$. This can be restated by dividing through by $(1 - \hat{x})$ and rearranging terms to obtain $w(1 - x_R) > (1 + r^*)$, which implies that the expected benefit of a dollar lent to a risky borrower must be greater than the cost per dollar. Given this assumption, we will also need the numerator of (A.4) to be negative, which implies that individuals must place enough value on future periods,

$$\delta > \frac{1 + z}{w(1 - x_R)(1 - \hat{x})}.$$  \hspace{1cm} (A.5)

Finally, for $T$ to be positive, the first term in (12) must be greater than one:

$$\ln \left( \frac{[1 + z - w\delta(1 - x_R)(1 - \hat{x})](1 - \delta(1 - \gamma))}{\gamma[1 + z - w(1 - x_R)(1 - \hat{x})]} \right) > 1$$

$$\Leftrightarrow \ln \left( \frac{[1 + z - w\delta(1 - x_R)(1 - \hat{x})](1 - \delta(1 - \gamma))}{\gamma[1 + z - w(1 - x_R)(1 - \hat{x})]} \right) < \ln \delta$$

$$\Leftrightarrow \frac{[1 + z - w\delta(1 - x_R)(1 - \hat{x})](1 - \delta(1 - \gamma))}{\gamma[1 + z - w(1 - x_R)(1 - \hat{x})]} < \delta$$

$$\Leftrightarrow (1 - \hat{x})[1 + z - w\delta(1 - x_R)(1 - \hat{x})] > \delta \gamma[1 + z - w(1 - x_R)(1 - \hat{x})]$$

$$\Leftrightarrow \gamma > \frac{w\delta(1 - x_R)(1 - \hat{x}) - (1 + z)}{w\delta(1 - x_R)(1 - \hat{x})}. \hspace{1cm} (A.6)$$

The probability of return to the lending phase must be large enough for $T^* > 0$. A violation of this single assumption means that $T^* = 0$ will satisfy the incentive constraint, or that strategic default will not occur. The right hand side of (A.6) implies that the ratio of expected profits to expected revenue cannot be too high, or that $T^* = 0$ only for an individual who has much to gain from the borrowing relationship.

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