TOPOLOGY OF ALGEBRAIC VARIETIES

ABSTRACTS OF TALKS

• DONU ARAPURA: Nori's Hodge conjecture.

Nori's conjecture, which is not so well known, says that his category of motives embeds fully and faithfully into the category of mixed Hodge structures.

This should be viewed as a refinement of Deligne's absoluteness conjecture.

I want to explain the conjecture, and then explain how to prove the special case for the tensor subcategory generated by smooth affine curves, which contains things like semiabelian varieties.

• INGRID BAUER: Burniat surfaces, moduli spaces and topology.

Burniat surfaces were constructed by P. Burniat in 1966, but still nowadays they are not completely understood. These surfaces are surfaces of general type with $p_g(S) = q(S) = 0$ AND $2 \leq K_S^2 \leq 6$. While Burniat surfaces with $K_S^2 = 6$ form an irreducible connected component of the moduli space of surfaces of general type and they are even determined by their homotopy type, the situation gets more complicated for decreasing K_S^2. In fact, I will also comment on some pathologies of the moduli space of surfaces which were observed for the first time in nodal Burniat and extended nodal Burniat surfaces with $K_S^2 = 4$.

• ARNAUD BEAUVILLE: The Lüroth problem and the Cremona group.

The Lüroth problem asks whether every field K with $\mathbb{C} \subset K \subset \mathbb{C}(x_1, \ldots, x_n)$ is of the form $\mathbb{C}(y_1, \ldots, y_p)$. After a brief historical survey, I will recall the counter-examples found in the 70's; then I will describe a quite simple (and new) counter-example. Finally I will explain the relation with the study of the finite groups of birational automorphisms of $\mathbb{P}^3$.

• FREDERIC CAMPANA: Conjecture d'Abélianité pour les variétés Kaehlériennes compactes 'spéciales'.

Les variétés Kaehlériennes (compactes) spéciales sont celles ne dominant pas d''orbifoldes' de type général. Elles généralisent les courbes rationnelles et elliptiques en toutes dimensions, et sont antithetiques des variétés de type général. Toute variété Kaehlérienne compacte X se décompose canoniquement et fonctoriellement à l'aide d'une fibration (son 'coeur') en ses parties 'spéciales' (les fibres), et de type général (la 'base orbifolde'). Cette décomposition fournit conjecturalement un 'scindage' des propriétés de X (aux niveaux hyperbolique, arithmétique si X est projective, et topologique).

Par exemple, on conjecture que le groupe fondamental de X est virtuellement abélien si X est 'spéciale'. Cette conjecture est vraie si ce groupe fondamental est soit linéaire, soit résoluble. Nous démontrons (travail en commun avec B. Claudon) que c'est le cas si X est de dimension au plus 3 en utilisant des arguments métriques (Calabi-Yau orbifolde) et le programme des modèles minimaux en dimension 2.

• **FABRIZIO CATANESE: Special Galois coverings and the irreducibility of certain spaces of coverings of curves, with applications to the moduli space of curves.**

Special Galois coverings are e.g. cyclic or dihedral coverings, for which I will describe old and new results, and new examples, obtained together with Fabio Perroni and Michael Loenne.

In the case of curves I will show some irreducibility results for coverings of a fixed numerical type: in the cyclic case for smooth curves, and in the cyclic case of prime order for moduli-stable curves. In the dihedral case we have results in work in progress with Michael Loenne and Fabio Perroni: in the case where the genus of the base is 0, or in the case where the covering is étale. In this case our work ties in with some general asymptotical study done by Dunfield and Thurston.

One application of the cyclic case is the description of an irredundant irreducible decomposition for the singular locus of the compactified Moduli space of curves $\overline{\mathfrak{M}_g}$, extending the result of Cornalba for the open set $\mathfrak{M}_g$.

• **BRUNO KLINGLER: Symmetric differentials and Kähler groups.**

I will discuss the relation between rigidity properties for the fundamental group of a smooth projective variety X and the structure of symmetric holomorphic differentials on X.

• **MAHAN MJ: Three manifolds groups, Kähler groups and complex surfaces.**

Let $1 \to N \to G \to Q \to 1$ be an exact sequence of finitely presented groups, where Q is infinite and not virtually cyclic, and is the fundamental group of some closed 3-manifold. If G is Kahler, we show that Q contains as a finite index subgroup either a finite index subgroup of the 3-dimensional Heisenberg group, or the fundamental group of the Cartesian product of a closed oriented surface of positive genus and the circle.

If G is the fundamental group of a compact complex surface, we show that Q must contain the fundamental group of a Seifert-fibered three manifold as a finite index sub- group, and G contains as a finite index subgroup the fundamental group of an elliptic fibration. This is joint work with I. Biswas and H. Seshadri.

• STEFAN PAPADIMA: Diophantine geometry, representation theory and homology of the Johnson filtration.

I will present answers to questions raised by B. Farb and F. Cohen, concerning the homology of the second Johnson subgroup of Torelli groups. The approach is based on the representation theory of arithmetic groups, on affine tori and their Lie algebras. This is joint work with A. Dimca, R. Hain and A. Suciu.

• PIERRE PY: Kähler groups, real hyperbolic spaces and the Cremona group.

Starting from a classical theorem of Carlson and Toledo, we will discuss actions of fundamental groups of compact Khler manifolds on finite or infinite dimensional real hyperbolic spaces. We will see that such actions almost always (but not always) come from surface groups. We then give an application to the study of the Cremona group. This is a joint work with Thomas Delzant.

• ALEXANDRU SUCIU: Abelian Galois covers and rank one local systems.

The Galois covers of a connected, finite CW-complex X with group of deck transformations a fixed Abelian group admit a natural parameter space, which in the case of free abelian covers of rank r is simply the Grassmannian of r-planes in $H^1(X, \mathbb{Q})$. The Betti numbers of such covers are determined by the jump loci for homology with coefficients in rank 1 local systems on X, and the way these loci intersect with certain algebraic subgroups in the character group of $\pi_1(X)$. Under favorable circumstances, the finiteness of those Betti numbers is controlled by the jump loci of the cohomology ring of X. In this talk, I will discuss this circle of ideas, and give some new examples where such computations play a role, especially in the case when X is a smooth, quasi-projective complex variety.

• CLAIRE VOISIN: The decomposition theorem for families of K3 surfaces and Calabi-Yau hypersurfaces

The decomposition theorem for smooth projective morphisms $\pi : \mathcal{X} \to B$ says that $R\pi_*\mathbb{Q}$ decomposes as $\oplus R^i\pi_*\mathbb{Q}[-i]$. We describe simple examples where it is not possible to have such a decomposition compatible with cup-product, even after restriction to Zariski dense open sets of B. We prove however that this is always possible for families of $K3$ surfaces (after shrinking the base), and show how this result relates to a result by Beauville and the author on the Chow ring of $K3$ surfaces. We also prove that such a multiplicative decomposition isomorphism exists for Calabi-Yau hypersurfaces in $\mathbb{P}^n$.