ALGEBRAIC GEOMETRY GENOVA-NICE-TORINO

Torino, 8-9 February 2018

Schedule

Thusday 8 February

14:00-14:50 Arnaud Beauville (Nice): *Limits of the trivial bundle on a curve*

15:00-15:50 Arvid Perego (Genova): *Moduli spaces of sheaves on K3 surfaces and irreducible symplectic varieties*

Coffee break

16:20-17:10 Andreas Hoering (Nice): *Algebraic integrability of foliations with numerically trivial bundle*

17:20-18:10 Ada Boralevi (Politecnico Torino): *A construction of equivariant bundles on the space of symmetric forms*

Friday 9 February

9:10-10:00 Lea Terracini (Università Torino): *Embedding the Picard group inside the class group: the case of $\mathbb{Q}$-factorial complete toric varieties*

Coffee break

10:30-11:20 Stefano Vigni (Genova): *A p-adic Gross-Zagier formula for a rational elliptic curve*

11:30-12:20 Cristiana Bertolin (Università Torino): *The Brauer Group of 1-motives*

Abstracts

A. Beauville: *Limits of the trivial bundle on a curve*

Given a family of vector bundles E_t on a curve C, such that E_t is trivial for $t \neq 0$, what are the possible bundles E_0? I will give a complete answer in rank 2 when C is general or hyperelliptic; in that case all limit bundles are decomposable. Then I will give examples of indecomposable limit bundles on special curves.

C. Bertolin: *The Brauer Group of 1-motives*

Let S be a scheme. First we define the notion of Brauer group $\mathrm{Br}(X)$ for a locally ringed S-stack X and we construct an injective homomorphism from $\mathrm{Br}(X)$ to the cohomology group $H^2_{et}(X, G_m)$ which generalizes to stacks Grothendieck's injective homomorphism for schemes.

Let M be a 1-motive defined over a noetherian scheme S. We define the Brauer group $\mathrm{Br}(M)$ of M as the Brauer group of the Picard S-stack associated to M. Our main result is the following: if the base scheme S is normal and noetherian, and if the extension G underlying M satisfies the generalized Theorem of the Cube for a prime l distinct from the residue characteristics of S, then the l-primary component of the kernel of the homomorphism from $H^2_{et}(M, G_m) \to H^2_{et}(S, G_m)$ induced by the unit section of M, is contained in $\mathrm{Br}(M)$. If the 1-motive M reduces to an abelian scheme, we recover Hoobler's main result concerning the Brauer group of abelian schemes.

This is a joint work with Federica Galluzzi.

A. Boralevi: *A construction of equivariant bundles on the space of symmetric forms*

I will report on a recent joint work with D. Faenzi and P. Lella, where we give a new construction of indecomposable vector bundles on the space of symmetric forms of degree d in $n+1$ variables, which are equivariant for the action of the complex Lie group SL_{n+1}, and moreover admit an equivariant free resolution of length 2. For $n = 1$, we obtain new examples of indecomposable vector bundles of rank $d-1$ on $\mathbb{P}^d$, which are moreover equivariant for SL_2. The presentation matrix of these bundles attains the bound for matrices of linear forms of constant rank for that size and rank.

A. Hoering: *Algebraic integrability of foliations with numerically trivial bundle*

The Beauville-Bogomolov decomposition of smooth projective manifolds is a cornerstone of the classification of higher-dimensional varieties. For varieties with mild singularities (e.g. minimal models) the analytic tools used in the proof are not available, so one has to look for a different strategy. In this talk I will explain how the algebraic integrability of foliations enters the picture and how a positivity result for reflexive sheaves allows to extend the decomposition theorem to the singular setting. This is joint work with Thomas Peternell.

A. Perego: *Moduli spaces of sheaves on K3 surfaces and irreducible symplectic varieties*

Irreducible symplectic manifolds are one of the three building blocks of compact Kähler manifolds with numerically trivial canonical bundle (together with abelian varieties and Calabi-Yau manifolds), thanks to the Beauville-Bogomolov decomposition theorem. A recent result of A. Höring and T. Peternell has completed the extension of this decomposition theorem to singular projective varieties: irreducible symplectic varieties are the singular analogue of irreducible symplectic manifolds, and they are one of the building blocks of normal, projective varieties having canonical singularities and numerically trivial canonical bundle. In a recent joint work with A. Rapagnetta we prove that all moduli spaces of semistable sheaves over projective K3 surfaces (with respect to a generic polarization) are irreducible symplectic varieties, with the only exception of those isomorphic to symmetric products of K3 surfaces, and compute their Beauville form and Fujiki constant. Similar results are shown to hold for the Albanese fiber of moduli spaces of sheaves over Abelian surfaces.

L. Terracini: *Embedding the Picard group inside the class group: the case of $\mathbb{Q}$-factorial complete toric varieties*

Let X be a $\mathbb{Q}$-factorial complete toric variety over an algebraic closed field of characteristic 0. There is a canonical injection of the Picard group $\mathrm{Pic}(X)$ in the group $\mathrm{Cl}(X)$ of classes of Weil divisors. These two groups are finitely generated abelian groups; whilst the first one is a free group, the second one may have torsion. We investigate algebraic and geometrical conditions under which the image of $\mathrm{Pic}(X)$ in $\mathrm{Cl}(X)$ is contained in a free part of the latter group. This is a joint work with Michele Rossi.

S. Vigni: *A p-adic Gross-Zagier formula for a rational elliptic curve*

A celebrated result of Gross and Zagier gives a formula expressing the central value of the first derivative of the complex L-function associated with an elliptic curve E in terms of the Néron-Tate height of a distinguished Heegner point on E. The goal of this talk is to describe a formula of Gross-Zagier type for a certain p-adic L-function of E (where p is a prime number) that is defined, following a recipe proposed by Bertolini and Darmon, by means of distributions of Heegner points on modular (or, more generally, Shimura) curves. This is joint work (in progress) with Rodolfo Venerucci.