Workshop on Anosov representations: List of talks

Aussois, 29 August – 3 September 2021

1 Introduction to higher Teichmüller theory

Let S be a closed hyperbolic surface. This introductory talk will present the definitions of Hitchin representations and maximal representations of $\pi_1(S)$, and explain that they form connected components of the character variety that have strong analogies with the classical Teichmüller space of S. Historically, the notion of an Anosov representation was motivated by the understanding of these "higher Teichmüller spaces".

References

[1] M. BURGER, A. IOZZI, A. WIENHARD, *Higher Teichmüller Spaces: from* $\mathsf{SL}(2,\mathbb{R})$ *to other Lie groups*. Handbook of Teichmüller Theory IV, 2014.

[2] A. WIENHARD, *An invitation to higher Teichmüller theory*. Proceedings of the ICM 2018.

[3] M. B. POZZETTI, *Higher rank Teichmüller theories*. Séminaire Bourbaki, Exposé 1161, 2019.

2 Anosov representations: definition and first characterizations

This talk will be centered around the Lie groups $\mathsf{G} = \mathsf{SL}(n,\mathbb{R})$ and $\mathsf{G} = \mathsf{SO}(p,q)$ for $p,q \geq 1$. It will explain the original definition of Anosov representations into G for fundamental groups Γ of closed negatively-curved manifolds (due to Labourie), and its extension to all word hyperbolic groups Γ (by Guichard and Wienhard). It will give various characterizations of Anosov representations into G, in terms of boundary maps and singular values or eigenvalues.

References

[1] F. LABOURIE, *Anosov flows, surface groups and curves in projective space.* Inventiones Mathematicae, 2006.

[2] O. GUICHARD, A. WIENHARD, *Anosov representations: Domains of discontinuity and applications.* Inventiones Mathematicae, 2012. **[Part 1]**

[3] F. GUÉRITAUD , O. GUICHARD, F. KASSEL, A. WIENHARD, *Anosov representations and proper actions.* Geometry and Topology, 2017.

[4] M. KAPOVICH, B. LEEB, J. PORTI *Anosov subgroups: Dynamical and geometric characterizations.* European Journal of Mathematics, 2017.

[5] R. D. CANARY, *Anosov representations: informal lecture notes*, see
`http://www.math.lsa.umich.edu/∼canary/Anosovlecnotes.pdf`.

3 Anosov representations and uniform domination

This talk will explain that Anosov representations into $\mathsf{G} = \mathsf{SL}(n, \mathbb{R})$ can be characterized as *uniformly dominated* representations, or in other words as representations with a uniform gap in singular values (a result of Kapovich, Leeb and Porti). The talk will sketch the alternative proof given by Bochi, Potrie and Sambarino, from the point of view of dominated splittings.

References

[1] M. KAPOVICH, B. LEEB, J. PORTI *A Morse lemma for quasigeodesics in symmetric spaces and euclidean buildings.* Geometry and Topology, 2018.

[2] J. BOCHI, R. POTRIE, A. SAMBARINO, *Anosov representations and dominated splittings.* Journal of the European Mathematical Society, 2019.

[3] R. D. CANARY, *Anosov representations: informal lecture notes*, see
`http://www.math.lsa.umich.edu/∼canary/Anosovlecnotes.pdf`.

4 Anosov representations and $\mathbb{H}^{p,q-1}$-convex cocompactness

A representation of a discrete group into the rank-one Lie group $\mathsf{G} = \mathsf{SO}(p, 1)$ is Anosov if and only if it is convex cocompact. As a higher-rank generalization, this talk will explain how Anosov representations into $\mathsf{G} = \mathsf{SO}(p, q)$ can be characterized by a convex cocompactness property in the pseudo-Riemannian hyperbolic space $\mathbb{H}^{p,q-1}$, following Danciger, Guéritaud and Kassel. The case $q = 2$ (anti-de Sitter geometry) is due to Mess and to Barbot and Mérigot.

References

[1] T. BARBOT, Q. MÉRIGOT, *Anosov AdS representations are quasi-Fuchsian.* Groups, Geometry and Dynamics, 2012.

[2] J. DANCIGER, F. GUÉRITAUD, F. KASSEL, *Convex cocompactness in pseudo-Riemannian hyperbolic spaces.* Geometriae Dedicata, 2018.

5 Anosov representations and strong projective convex cocompactness

This talk will explain that discrete subgroups of $\mathsf{G} = \mathsf{SL}(n, \mathbb{R})$ acting cocompactly on some strictly convex open subset of $\mathbb{P}(\mathbb{R}^n)$ ("divisible convex set") are the images of Anosov representations, as follows from the work of Benoist. Generalizing this result as well as the results of talk 4, it will describe the close relations (established by Danciger, Guéritaud and Kassel, and independently Zimmer) between Anosov representations into $\mathsf{G} = \mathsf{SL}(n, \mathbb{R})$ and strongly convex cocompact actions on properly convex open subsets of $\mathbb{P}(\mathbb{R}^n)$.

References

[1] Y. BENOIST, *Convexes divisibles I*, in *Algebraic groups and arithmetic*, Tata Institute of Fundamental Research Studies in Mathematics, 2004.

[2] J. DANCIGER, F. GUÉRITAUD, F. KASSEL, *Convex cocompact actions in real projective geometry.* arXiv:1704.08711.

[3] A. ZIMMER, *Projective Anosov representations, convex cocompact actions, and rigidity.* Journal of Differential Geometry, to appear.

[4] F. KASSEL, *Geometric structures and representations of discrete groups*, Proceedings of the ICM 2018. **[Section 6]**

6 Examples of Anosov representations using Coxeter groups

The results of talks 4 and 5 allow to construct new examples of Anosov representations, for discrete groups that are not necessarily surface groups or free groups. This talk will explain how to construct Anosov representations of any word hyperbolic Coxeter group using convex cocompactness in $\mathbb{H}^{p,q-1}$ or $\mathbb{P}(\mathbb{R}^n)$, following Danciger, Guéritaud, Kassel, Lee and Marquis.

References

[1] J. DANCIGER, F. GUÉRITAUD, F. KASSEL, *Convex cocompactness in pseudo-Riemannian hyperbolic spaces.* Geometriae Dedicata, 2018. **[Section 8]**

[2] J. DANCIGER, F. GUÉRITAUD, F. KASSEL, G.-S. LEE, L. MARQUIS *Convex cocompactness for Coxeter groups.* arXiv:2102.02757.

7 Cocompact domains of discontinuity I

This talk will explain how Anosov representations into $\mathsf{G} = \mathsf{SO}(p,q)$ give rise to cocompact domains of discontinuity in spaces of isotropic subspaces of $\mathbb{R}^{p,q}$, following Frances (case $q = 2$) and Guichard and Wienhard (general case). This can be used to construct cocompact domains of discontinuity for Anosov representations into more general Lie groups, using embeddings into $\mathsf{SO}(p,q)$.

References

[1] C. FRANCES, *Lorentzian Kleinian groups.* Commentarii Mathematici Helvetici, 2005.

[2] O. GUICHARD, A. WIENHARD, *Anosov representations: Domains of discontinuity and applications.* Inventiones Mathematicae, 2012. **[Part 2]**

8 Cocompact domains of discontinuity II

As a generalization of talk 7, this talk will survey the general theory of Kapovich, Leeb and Porti on the existence and construction of cocompact domains of discontinuity in G/Q for P-Anosov representations into G, where P and Q are suitable parabolic subgroups of G.

References

[1] M. KAPOVICH, B. LEEB, AND J. PORTI, *Dynamics on flag manifolds: domains of proper discontinuity and cocompactness.* Geometry and Topology, 2018.

[2] D. DUMAS AND A. SANDERS, *Geometry of compact complex manifolds associated to generalized quasi-Fuchsian representations.* Geometry and Topology, to appear. **[Section 3]**

[3] F. STECKER, N. TREIB, *Domains of discontinuity in oriented flag manifolds.* arXiv:1806.04459.

9 The pressure metric

This talk will explain how thermodynamic formalism can be used to define a mapping-class-group-invariant Riemannian metric ("pressure metric") on the space of Anosov representations, following Bridgeman, Canary, Labourie and Sambarino.

References

[1] M. BRIDGEMAN, R. CANARY, F. LABOURIE, A. SAMBARINO, *The pressure metric for convex representations.* Geometric and Functional Analysis, 2015.

[2] M. BRIDGEMAN, R. CANARY, A. SAMBARINO, *An introduction to pressure metrics for higher Teichmüller spaces.* Ergodic Theory and Dynamical Systems, 2018.

[3] M. BRIDGEMAN, R. CANARY, F. LABOURIE, A. SAMBARINO, *Simple root flows for Hitchin representations.* Geometriae Dedicata, 2018.

10 The relatively hyperbolic case

This final talk will discuss possible extensions of the notion of an Anosov representation to relatively hyperbolic groups, following the work of Kapovich and Leeb and that of Zhu.

References

[1] M. KAPOVICH, B. LEEB, *Relativizing characterizations of Anosov subgroups, I.* arXiv:1807.00160.

[2] F. ZHU, *Relatively dominated representations.* Annales de l'Institut Fourier, to appear.

This workshop is funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (ERC starting grant DiGGeS, grant agreement No 715982), and by the Louis D. foundation.