A microwave realization of artificial graphene

Matthieu Bellec

Laboratoire de Physique de la Matière Condensée,
CNRS & University of Nice-Sophia Antipolis, Nice, France

Journée de la physique niçoise

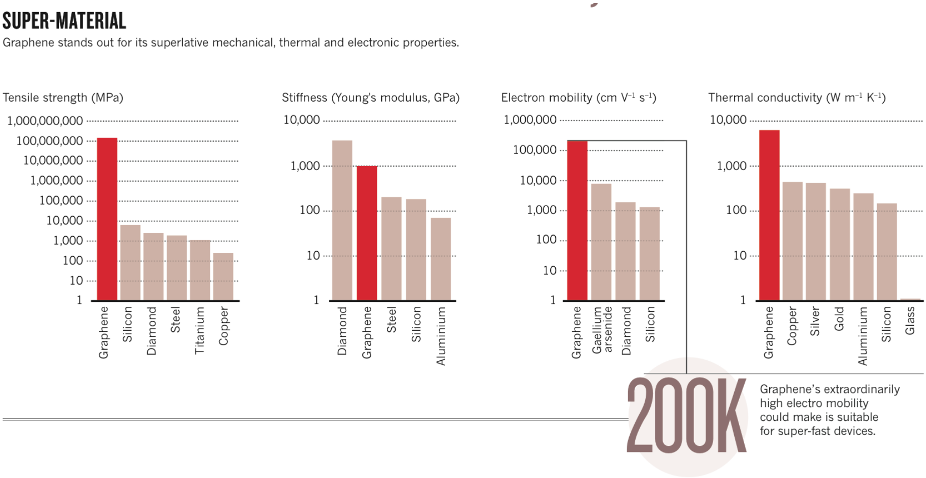
Sophia Antipolis, June 20th, 2014

Co-workers

Ulrich Kuhl
LPMC

Fabrice Mortessagne
LPMC

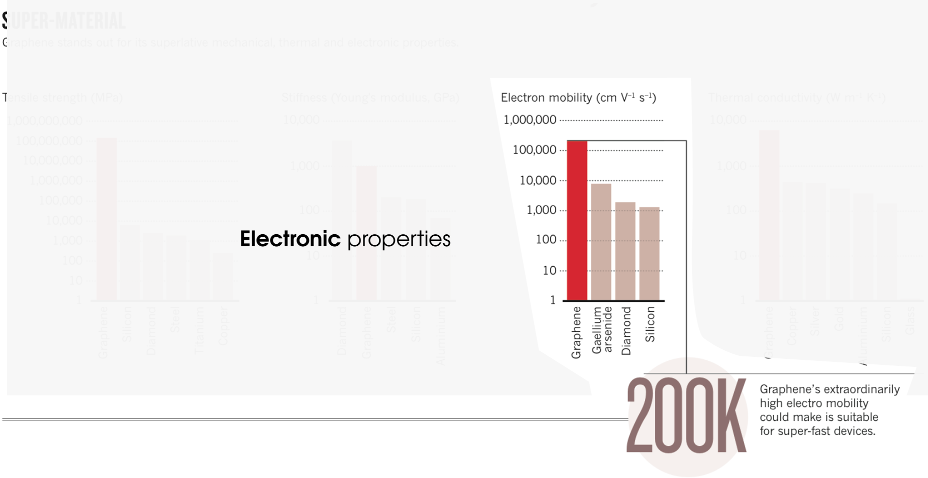
Gilles Montambaux
LPS, Orsay



*Outlook, Nature **483**, S29 (2012)*

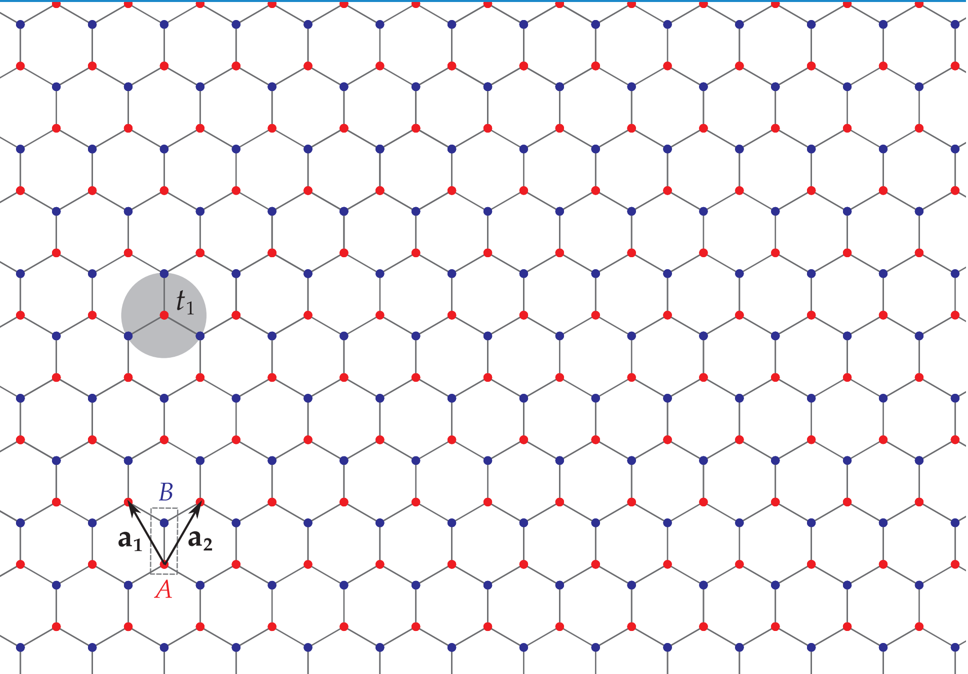
Amazing graphene

Electronic properties

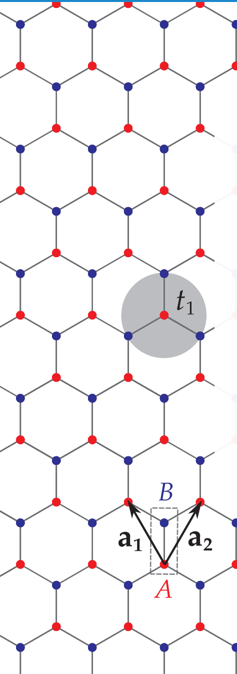


*Outlook, Nature **483**, S29 (2012)*

Tight-binding Hamiltonian in **regular** honeycomb lattice



Tight-binding Hamiltonian in **regular** honeycomb lattice



- Bloch states representation

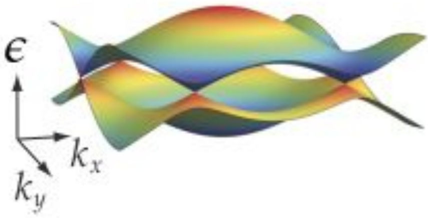
$$|\Psi_{\mathbf{k}}\rangle = \frac{1}{\sqrt{N}} \sum_j (\lambda_A |\phi_j^A\rangle + \lambda_B |\phi_j^B\rangle) e^{i\mathbf{k}\cdot\mathbf{R_j}}$$

- Effective Bloch Hamiltonian in (A, B) basis

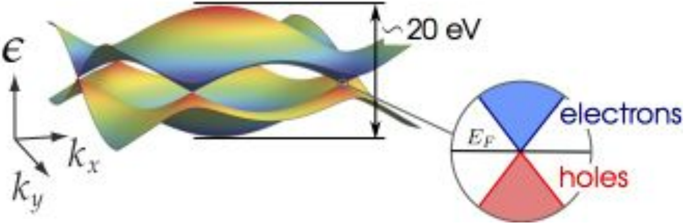
$$\mathcal{H}_{eff}^B = -t_1 \begin{pmatrix} 0 & f(\mathbf{k}) \\ f^*(\mathbf{k}) & 0 \end{pmatrix} \quad \text{with} \quad f(\mathbf{k}) = 1 + e^{-i\mathbf{k}\cdot\mathbf{a_1}} + e^{-i\mathbf{k}\cdot\mathbf{a_2}}$$

- Dispersion relation : $\epsilon(\mathbf{k}) = \pm t_1 |f(\mathbf{k})|$

Dispersion relation

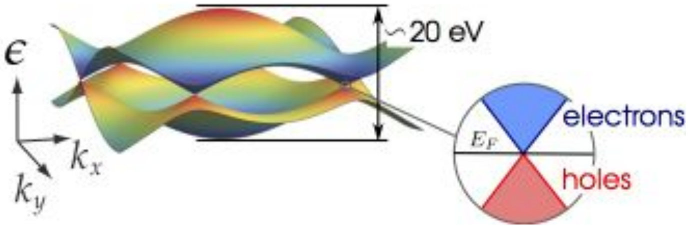


Dispersion relation



Low energy expension $\Rightarrow$ **Dirac Hamiltonian** for massless particle with $v_f \simeq c/300$

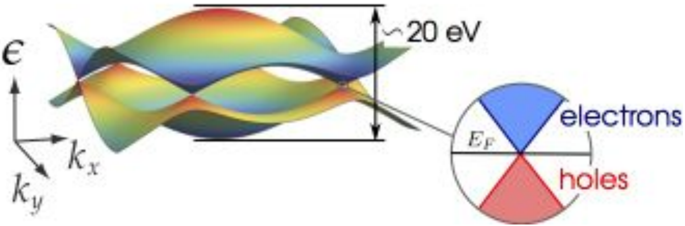
Dispersion relation



Low energy expension $\Rightarrow$ **Dirac Hamiltonian** for massless particle with $v_f \simeq c/300$

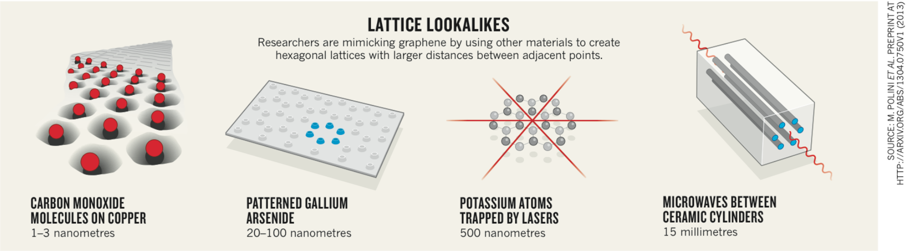
- Applications : very high charge carrier mobility

Dispersion relation



Low energy expension $\Rightarrow$ **Dirac Hamiltonian** for massless particle with $v_f \simeq c/300$

- Applications : very high charge carrier mobility
- Fundamental interests *e.g.* Klein tunneling



PHYSICS

Graphene knock-offs probe ultrafast electronics

Honeycomb lattices in different materials enable experiments impossible in the real thing.

E.S. Reich, Nature ***497****, 422 (2013)*

"The objective of creating these artificial graphene-like lattices is to produce **new systems** that have properties that graphene does not have." A. Castro Neto

Graphene knock-offs probe ultrafast electronics

Honeycomb lattices in different materials enable experiments impossible in the real thing.

*E.S. Reich, Nature **497**, 422 (2013)*

Typical experimental set-up with microwave lattices

Formal analogy between the **Schrödinger** and the **Helmholtz** equations

Free particle

$$[-\Delta + V(\vec{r})]\psi(\vec{r}) = E\psi(\vec{r})$$

Varying potential V

Microwave cavity

$$[-\Delta + (1 - \epsilon(\vec{r}))k^2]\psi(\vec{r}) = k^2\psi(\vec{r})$$

Varying permittivity ϵ

Formal analogy between the **Schrödinger** and the **Helmholtz** equations

Free particle

$$[-\Delta + V(\vec{r})]\psi(\vec{r}) = E\psi(\vec{r})$$

Varying potential V

Microwave cavity

$$[-\Delta + (1-\epsilon(\vec{r}))k^2]\psi(\vec{r}) = k^2\psi(\vec{r})$$

Varying permittivity ϵ

TM modes : $\psi(\vec{r}) = E_z(\vec{r}) \rightarrow$ energy everywhere (continuum state)

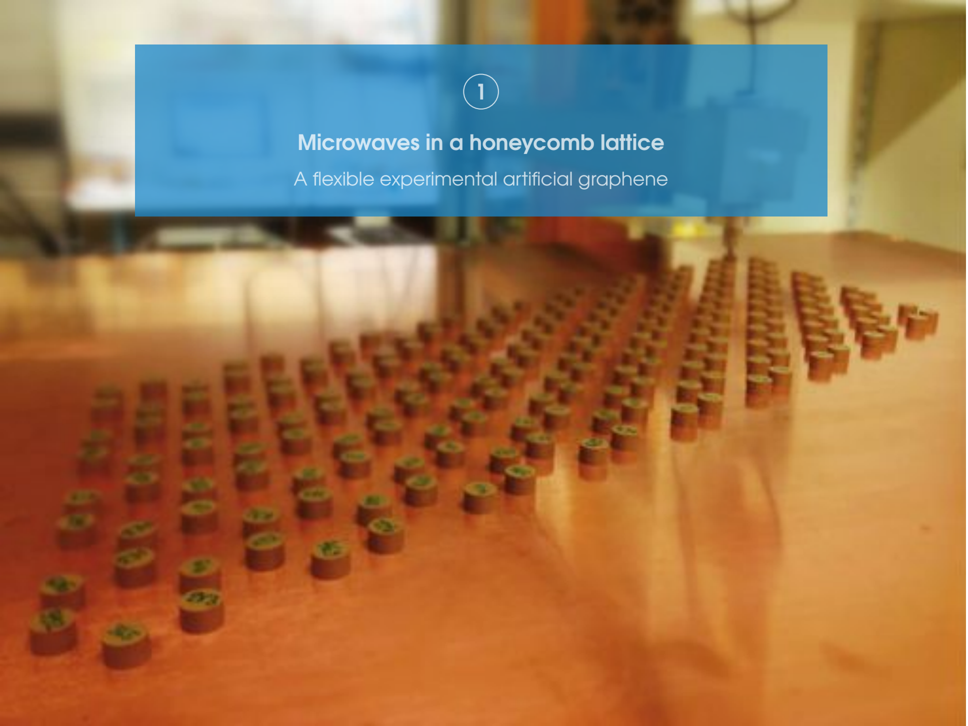
TE modes : $\psi(\vec{r}) = B_z(\vec{r}) \rightarrow$ energy confined inside (bound state)

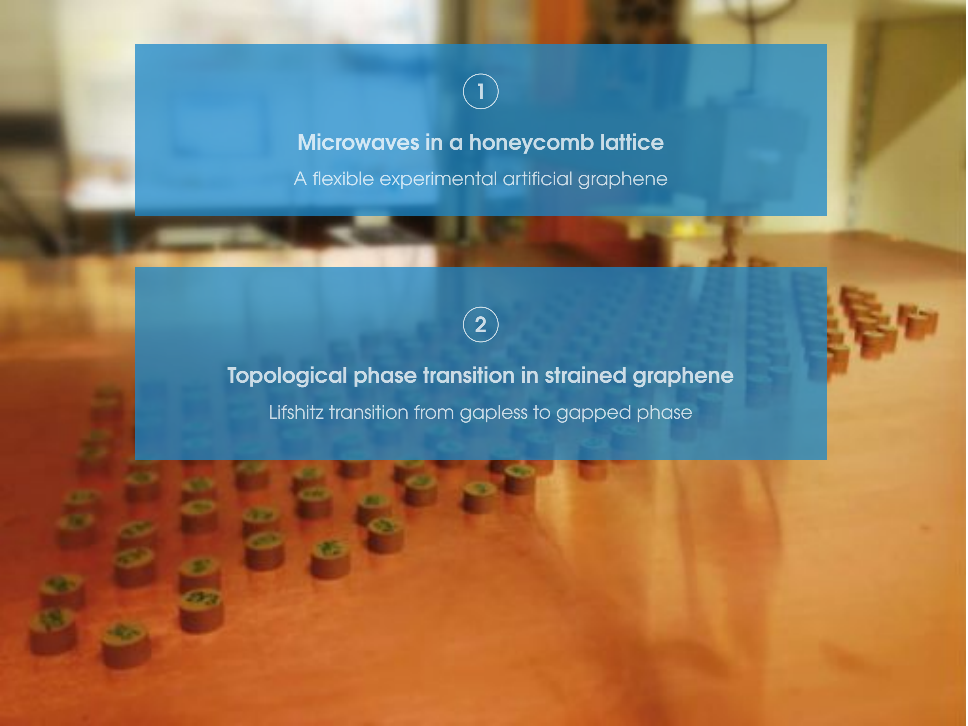
Attractive implementation to perform
quantum analogue measurements

1

Microwaves in a honeycomb lattice

A flexible experimental artificial graphene





1

Microwaves in a honeycomb lattice

A flexible experimental artificial graphene

2

Topological phase transition in strained graphene

Lifshitz transition from gapless to gapped phase

1

Microwaves in a honeycomb lattice

A flexible experimental artificial graphene

2

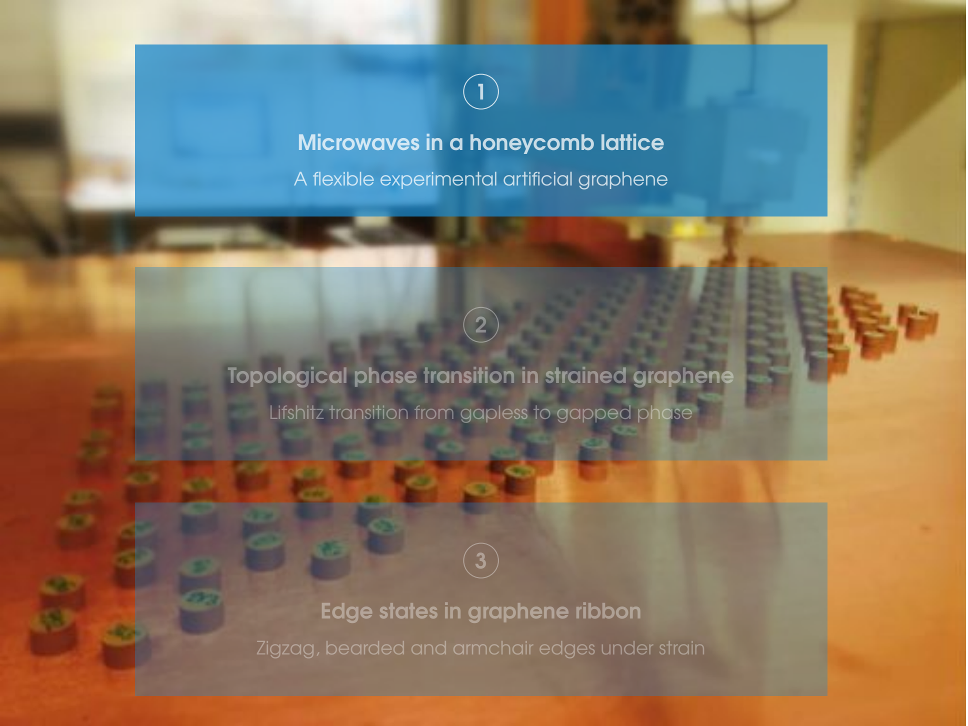
Topological phase transition in strained graphene

Lifshitz transition from gapless to gapped phase

3

Edge states in graphene ribbon

Zigzag, bearded and armchair edges under strain



1

Microwaves in a honeycomb lattice

A flexible experimental artificial graphene

2

Topological phase transition in strained graphene

Lifshitz transition from gapless to gapped phase

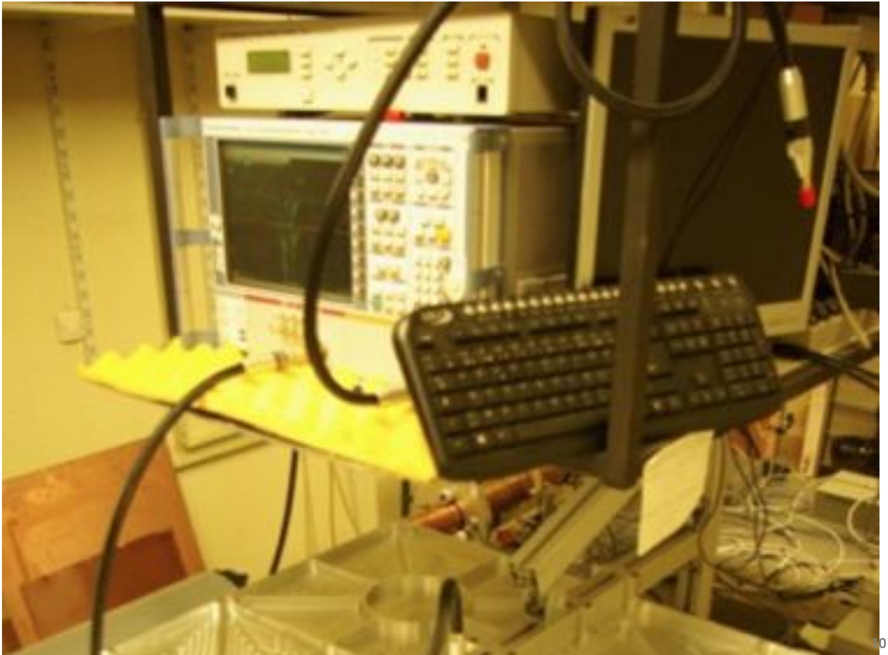
3

Edge states in graphene ribbon

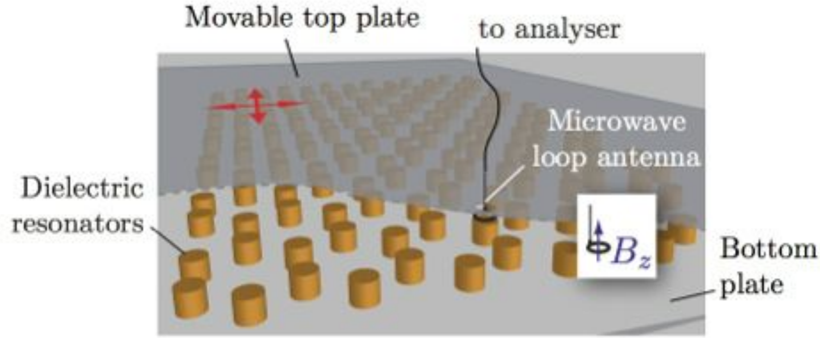
Zigzag, bearded and armchair edges under strain

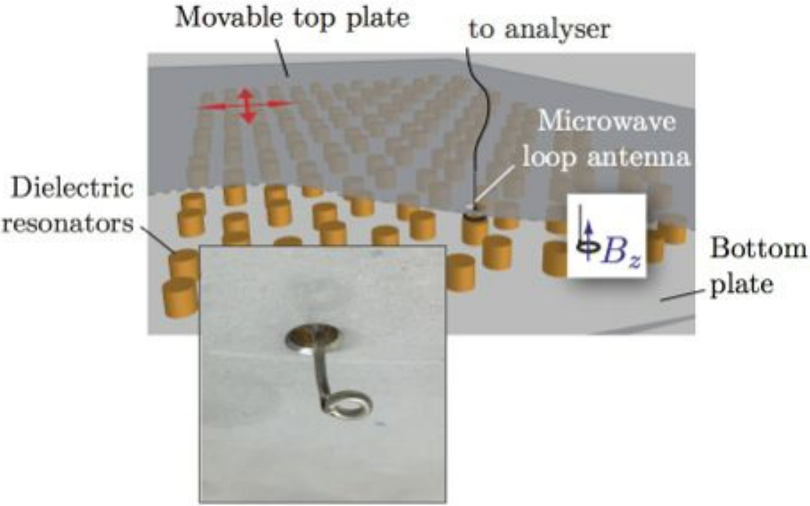

The experimental set-up in reality...

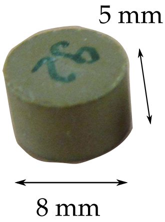
Experimental setup



Experimental setup

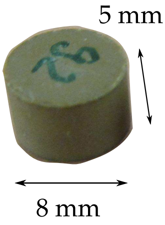


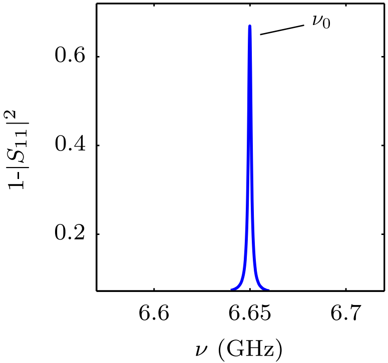

Movable top plate
to analyser
Microwave
loop antenna
Dielectric
resonators
B_z
Bottom
plate



- Dielectric cylinder : $n = 6$

An artificial atom

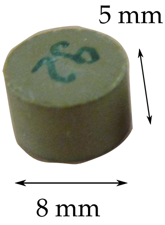


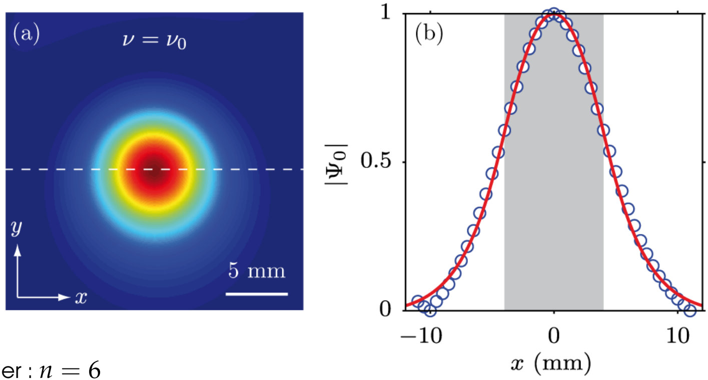


- Dielectric cylinder : $n = 6$
- We measure the **reflected** signal $S_{11}(\nu)$. At $\nu = \nu_0 \to 1 - |S_{11}(\nu_0)|^2 \simeq \frac{2\sigma}{\Gamma}|\Psi_0(\mathbf{r}_1)|^2$

  Γ^{-1} : lifetime ($\Gamma \sim 10$ MHz)
  σ : antenna coupling (weak and constant)

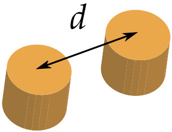
An artificial atom

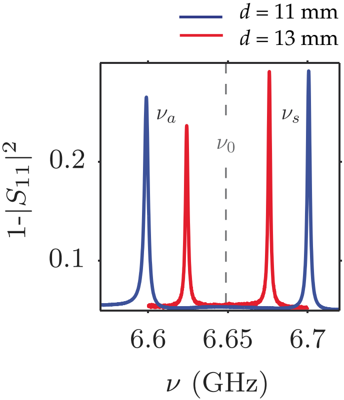




- Dielectric cylinder : $n = 6$
- We measure the **reflected** signal $S_{11}(\nu)$. At $\nu = \nu_0 \to 1 - |S_{11}(\nu_0)|^2 \simeq \frac{2\sigma}{\Gamma}|\Psi_0(\mathbf{r}_1)|^2$

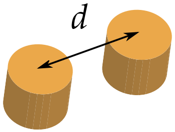
  Γ^{-1} : lifetime ($\Gamma \sim 10$ MHz)
  σ : antenna coupling (weak and constant)
- Most of the energy is confined in the disc (J_0) and spreads evanescently (K_0)

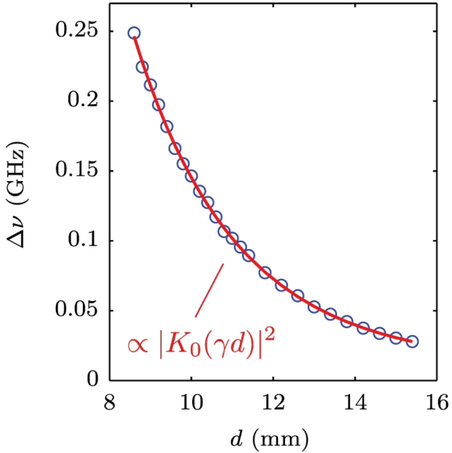




- The frequency splitting $\Delta\nu(d)$ gives the **coupling strength** $t_1(d) = \Delta\nu(d)/2$

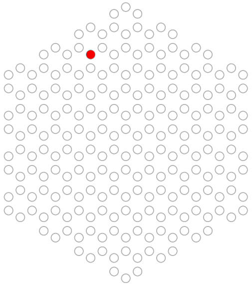
Coupling between two discs

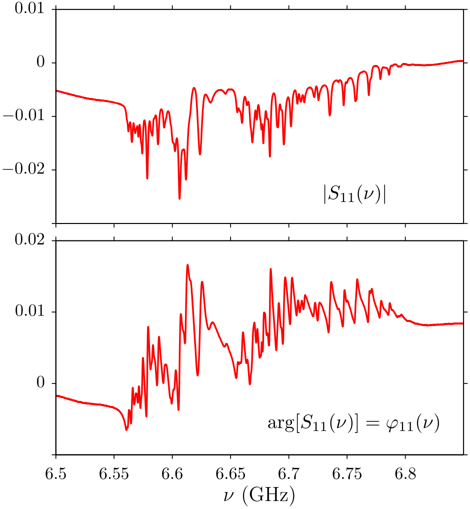


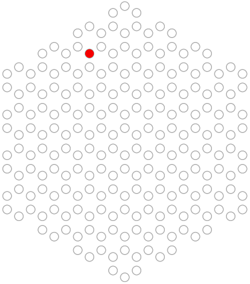


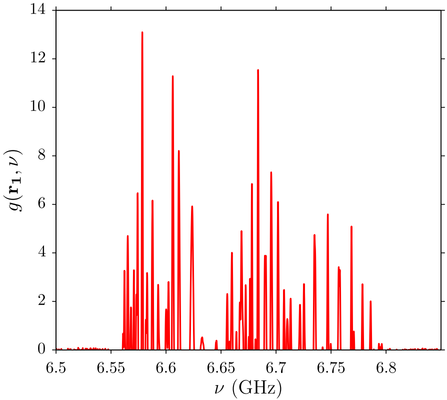
- The frequency splitting $\Delta\nu(d)$ gives the **coupling strength** $t_1(d) = \Delta\nu(d)/2$
- $|t_1(d)| = \alpha |K_0(\gamma d/2)|^2 + \delta$

Experimental (local) density of states





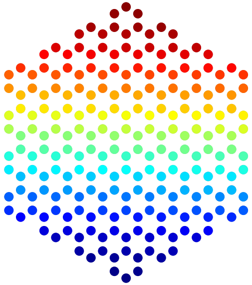


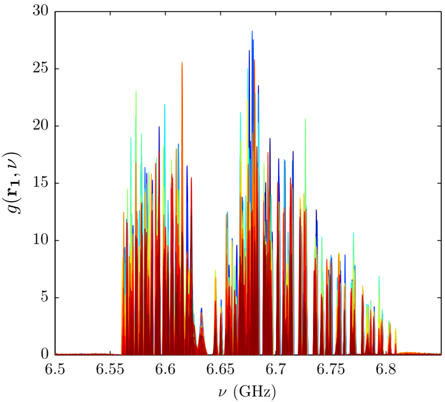


We have a direct acces to the **LDOS**

$$g(\mathbf{r}_1, \nu) = \frac{|S_{11}(\nu)|^2}{\langle |S_{11}|^2 \rangle_\nu} \varphi'_{11}(\nu) \sim -\frac{\sigma}{\Gamma \langle |S_{11}|^2 \rangle_\nu} \sum_n |\Psi_n(\mathbf{r}_1)|^2 \delta(\nu - \nu_n)$$

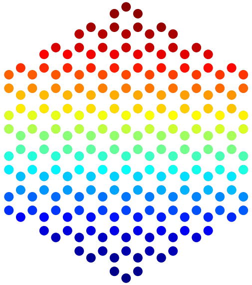
Experimental (local) density of states

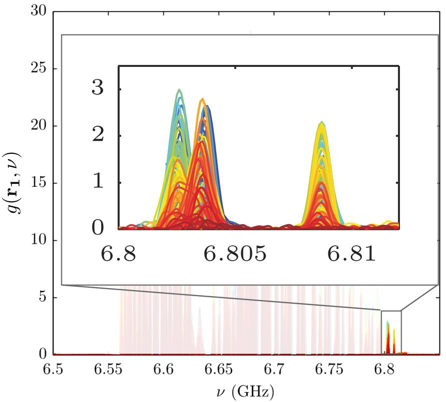




We have a direct acces to the **LDOS**

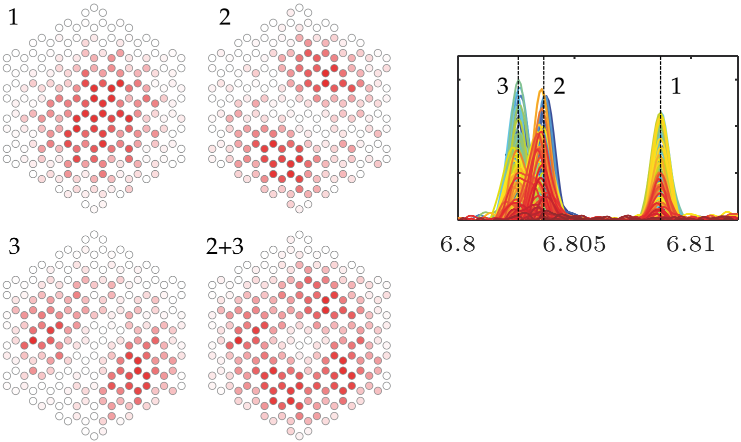
$$g(\mathbf{r}_1, \nu) = \frac{|S_{11}(\nu)|^2}{\langle |S_{11}|^2 \rangle_\nu} \varphi'_{11}(\nu) \sim -\frac{\sigma}{\Gamma \langle |S_{11}|^2 \rangle_\nu} \sum_n |\Psi_n(\mathbf{r}_1)|^2 \delta(\nu - \nu_n)$$





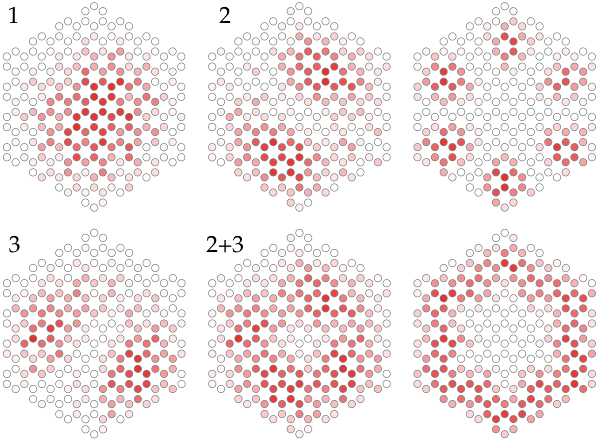
We have a direct acces to the **LDOS**

$$g(\mathbf{r}_1,\nu) = \frac{|S_{11}(\nu)|^2}{\langle |S_{11}|^2\rangle_\nu}\varphi'_{11}(\nu) \sim -\frac{\sigma}{\Gamma\,\langle |S_{11}|^2\rangle_\nu} \sum_n |\Psi_n(\mathbf{r}_1)|^2\delta(\nu-\nu_n)$$



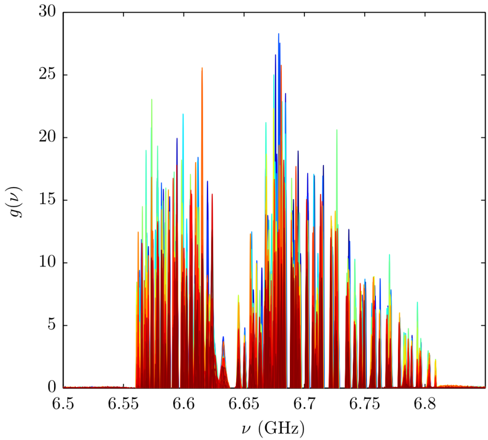
One can get the **wavefunction** associated to the eigenfrequency ν_n

Experimental (local) density of states

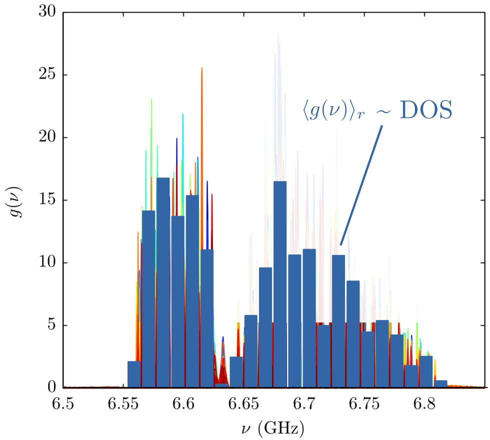


One can get the **wavefunction** associated to the eigenfrequency ν_n

Experimental (local) density of states

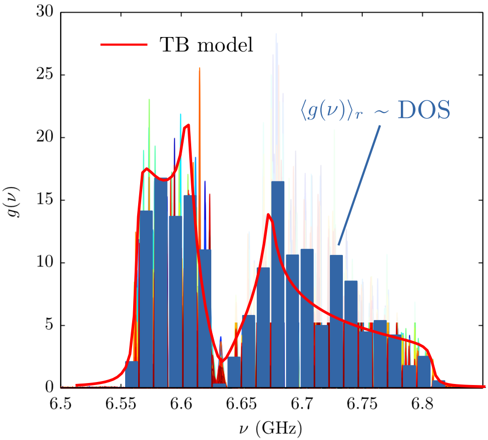


- By averaging $g(\mathbf{r}_i, \nu)$ over all the antenna positions $\mathbf{r}_i$, we obtain the **DOS**

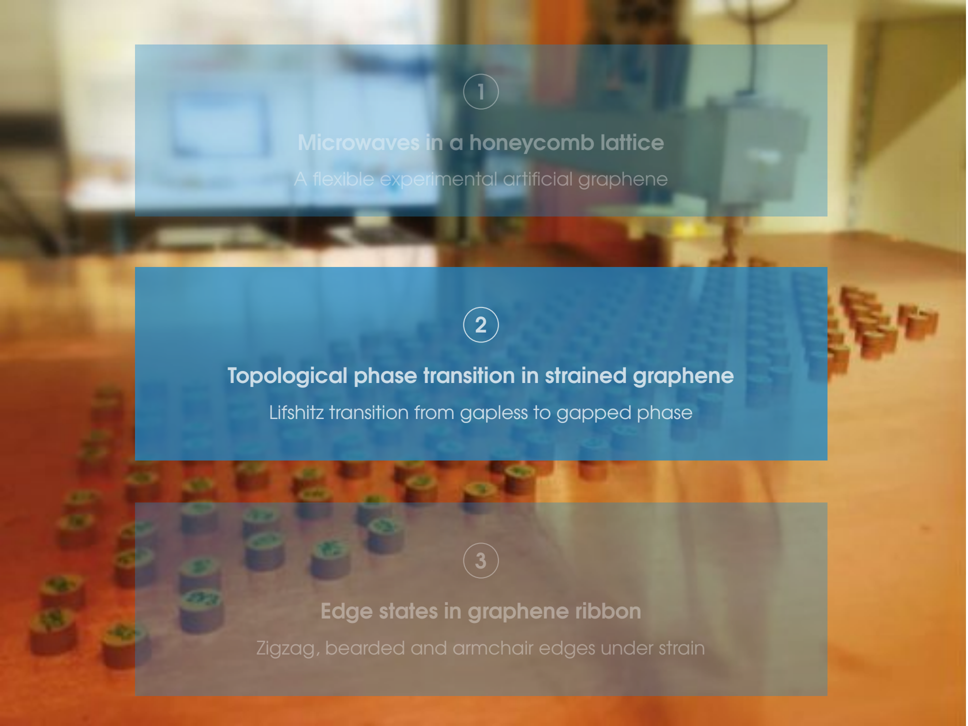


- By averaging $g(\mathbf{r}_i, \nu)$ over all the antenna positions $\mathbf{r}_i$, we obtain the **DOS**

Experimental (local) density of states



- By averaging $g(\mathbf{r}_i, \nu)$ over all the antenna positions $\mathbf{r}_i$, we obtain the **DOS**
- **Tight-binding compatible**. Main features have been taken into account Dirac shift, band asymmetry $\rightarrow$ **next n.n. couplings**.

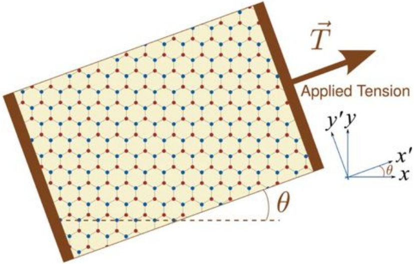

1
Microwaves in a honeycomb lattice
A flexible experimental artificial graphene
2
Topological phase transition in strained graphene
Lifshitz transition from gapless to gapped phase
3
Edge states in graphene ribbon
Zigzag, bearded and armchair edges under strain

- Mechanical response & electronic properties $\rightarrow$ **tunable** electronic properties

Graphene under strain – **Motivations**

- Mechanical response & electronic properties $\rightarrow$ **tunable** electronic properties
- Induce a robust, clean **bulk spectral gap** in graphene

- Mechanical response & electronic properties $\longrightarrow$ **tunable** electronic properties
- Induce a robust, clean **bulk spectral gap** in graphene
- 20% deformations required to open a gap

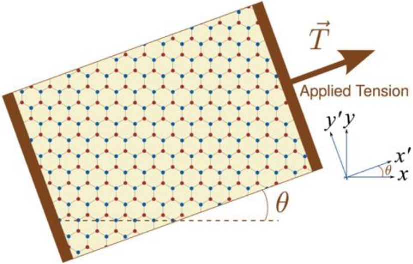


$\Rightarrow$ Uni-axial strain ineffective to achieve bulk gapped graphene.

Peirera et al., Phys. Rev. B ***80*** *045401 (2009)*

Graphene under strain – **Motivations**

- Mechanical response & electronic properties $\longrightarrow$ **tunable** electronic properties
- Induce a robust, clean **bulk spectral gap** in graphene
- 20% deformations required to open a gap



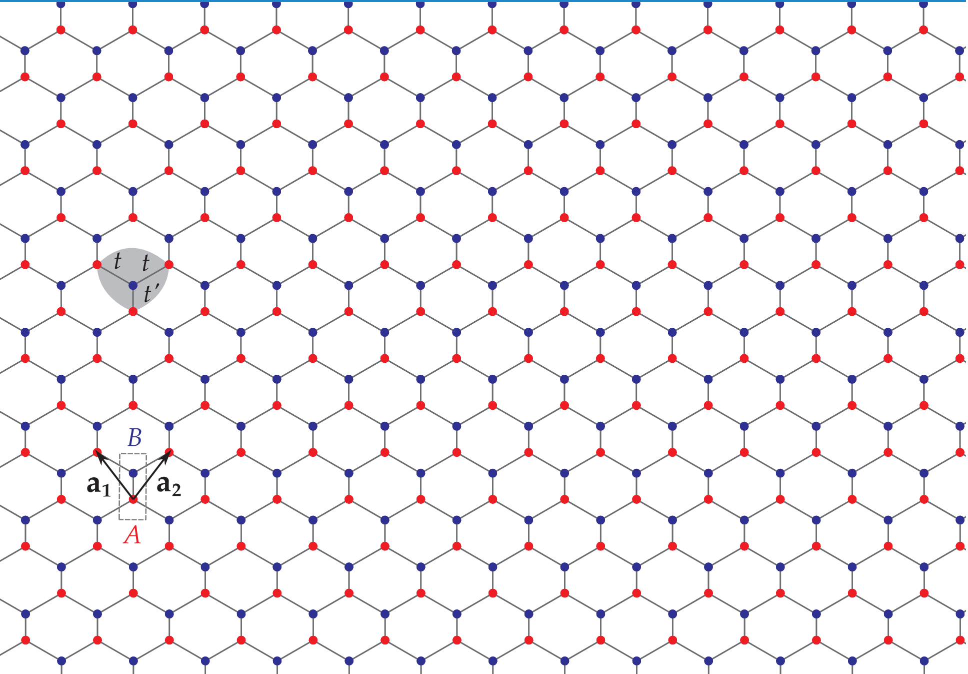
$\Rightarrow$ Uni-axial strain ineffective to achieve bulk gapped graphene.

*Peirera et al., Phys. Rev. B **80** 045401 (2009)*

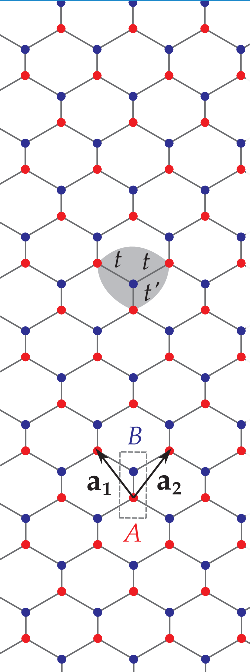
- Strain in **artificial** systems : $\alpha = \dfrac{a}{t}\dfrac{\partial t}{\partial a}$ with a site separation and t coupling term

$$\alpha_{microwave} \simeq 2\alpha_{graphene}$$

TB Hamiltonian in **uni-axial strained** honeycomb lattice



TB Hamiltonian in **uni-axial strained** honeycomb lattice

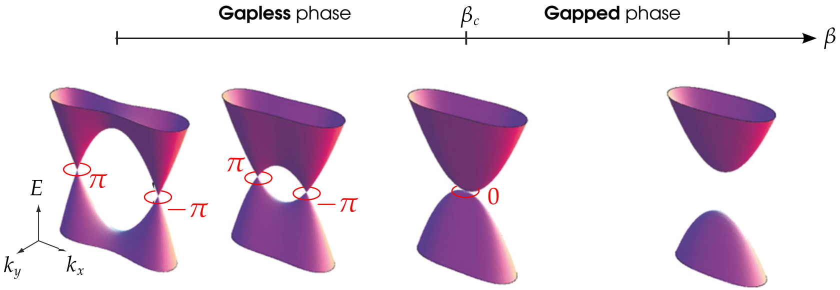


- **Anisotropy** parameter : $\beta = t'/t$
- Bloch Hamiltonian

$$\mathcal{H}^{B}_{eff} = -t|f(\mathbf{k})| \begin{pmatrix} 0 & e^{-i\phi(\mathbf{k})} \\ e^{i\phi(\mathbf{k})} & 0 \end{pmatrix}$$

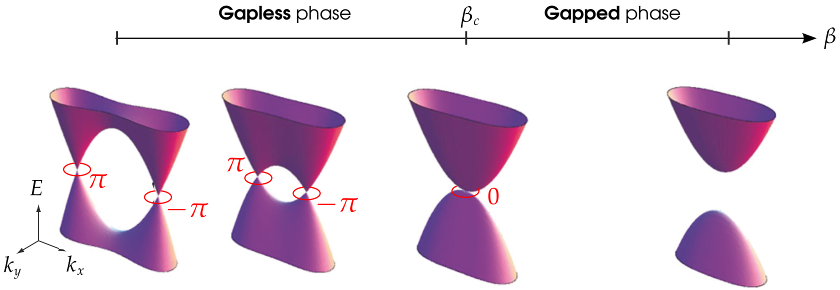
  with $f(\mathbf{k}) = \beta + e^{-i\mathbf{k}\cdot\mathbf{a}_1} + e^{-i\mathbf{k}\cdot\mathbf{a}_2}$

- Dispersion relation : $\epsilon(\mathbf{k}) = \pm t|f(\mathbf{k})|$
- Eigenstates : $\Psi_{\mathbf{k},\pm}(\mathbf{r}) = \dfrac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi(\mathbf{k})} \\ \pm 1 \end{pmatrix} e^{i\mathbf{k}\cdot\mathbf{r}}$
- Berry phase : $\gamma = \dfrac{1}{2} \oint d\mathbf{k}\, \nabla_{\mathbf{k}}\phi(\mathbf{k})$



- Berry phase $\pm\pi$ vanishes to 0 $\Rightarrow$ **Topological** phase transition
- Dirac points move, merge (at $\beta = \beta_c = 2$) and annihilate

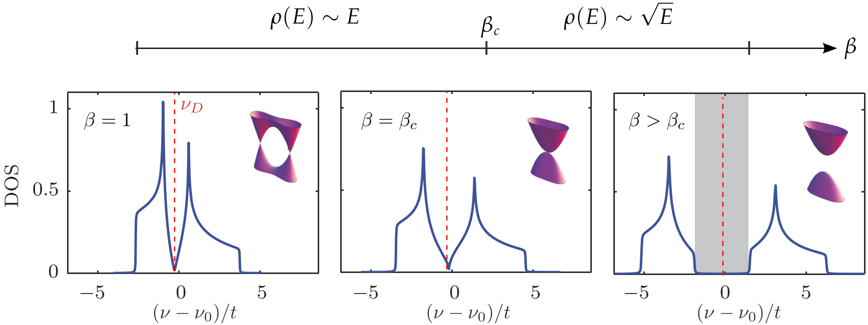
*G. Montambaux et al., Eur. Phys. J. B **72**, 509 (2009)*



- Berry phase $\pm\pi$ vanishes to 0 $\Rightarrow$ **Topological** phase transition
- Dirac points move, merge (at $\beta = \beta_c = 2$) and annihilate
- Phase transition from gapless to gapped phase (Lifshitz transition)

G. Montambaux et al., Eur. Phys. J. B ***72****, 509 (2009)*

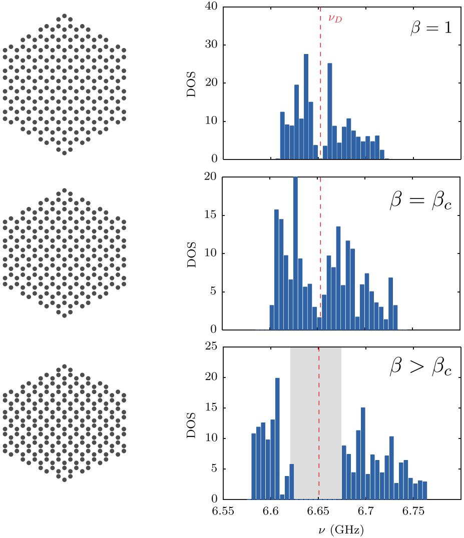
Topological phase transition



- Berry phase $\pm\pi$ vanishes to 0 $\Rightarrow$ **Topological** phase transition
- Dirac points move, merge (at $\beta = \beta_c = 2$) and annihilate
- Phase transition from gapless to gapped phase (Lifshitz transition)

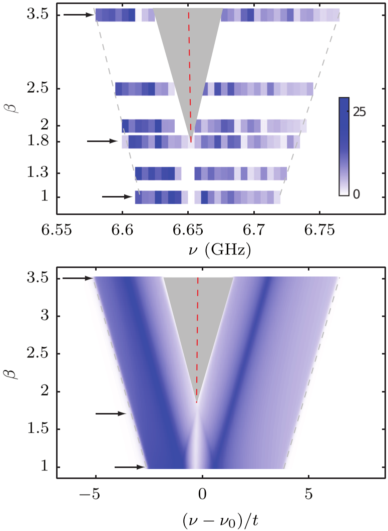
*G. Montambaux et al., Eur. Phys. J. B **72**, 509 (2009)*

Observation of the topological phase transition



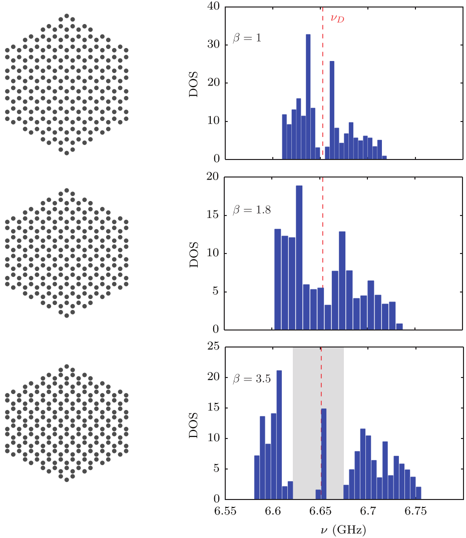
- **Transition** at $\beta = \beta_c$ and bandgap opening

Observation of the topological phase transition



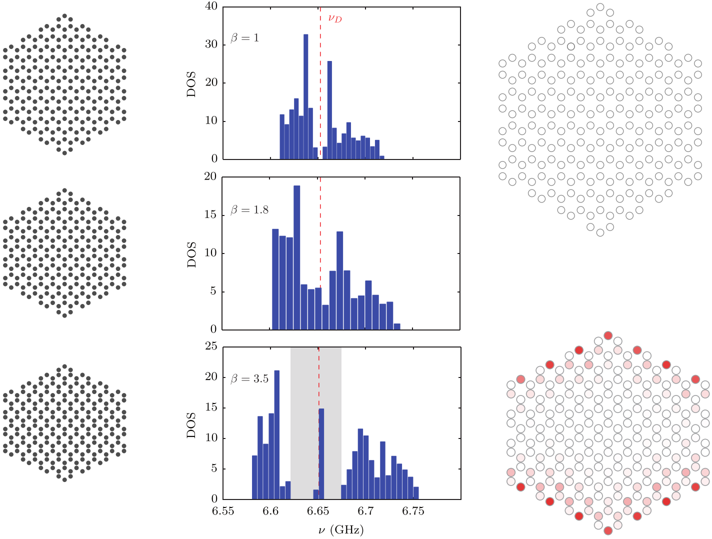
- **Transition** at $\beta = \beta_c$ and bandgap opening

Observation of the topological phase transition

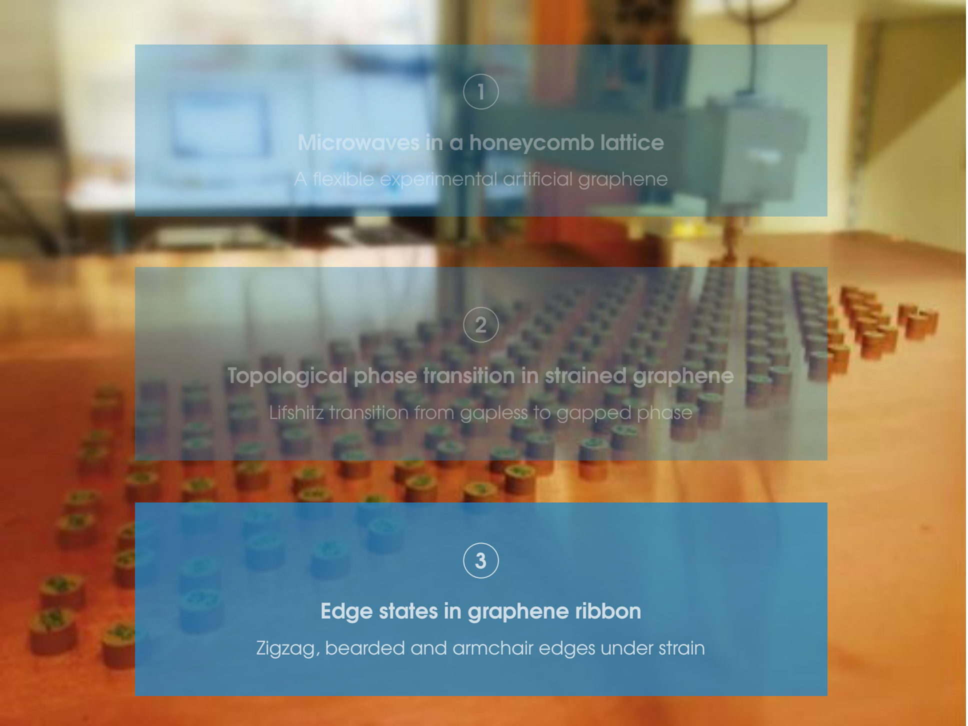


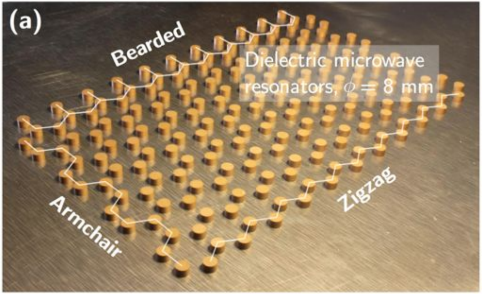
- **Transition** at $\beta = \beta_c$ and bandgap opening
- Presence of **edge** states at Dirac frequency

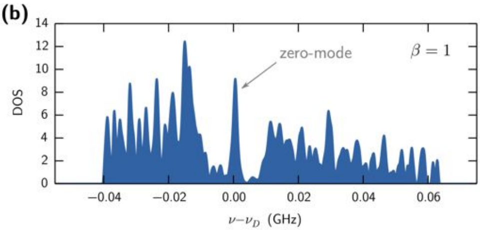
Observation of the topological phase transition



- **Transition** at $\beta = \beta_c$ and bandgap opening
- Presence of **edge** states at Dirac frequency


1
Microwaves in a honeycomb lattice
A flexible experimental artificial graphene
2
Topological phase transition in strained graphene
Lifshitz transition from gapless to gapped phase
3
Edge states in graphene ribbon
Zigzag, bearded and armchair edges under strain





- Edge states generally appear.

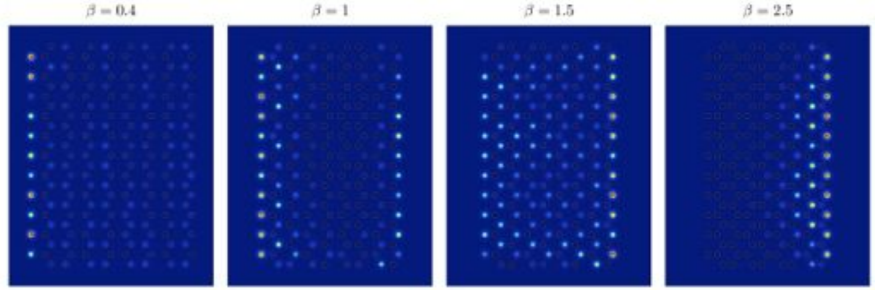
  Absence of **armchair** edge states is an **exception** !

- How to figure it out ?
  $\longrightarrow$ **Zak phase**

  S. Ryu and Y. Hatsugai, PRL (2002)

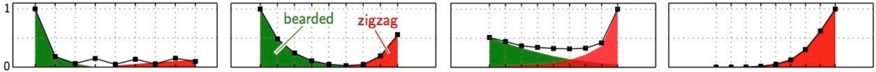
  P. Delplace et al., PRB (2011)

Experimental wavefunction intensities at ν_D



- β allows to control and manipulate edge states
- Localization length ξ depends on β

Zigzag and bearded edge states



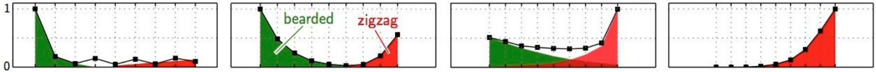
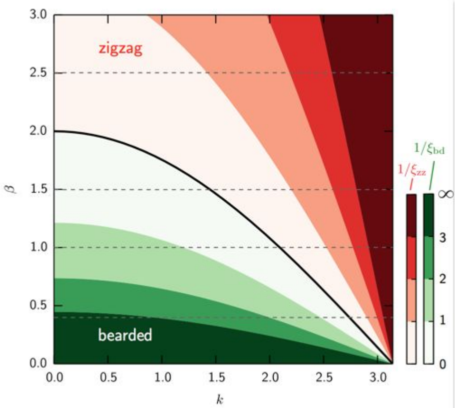
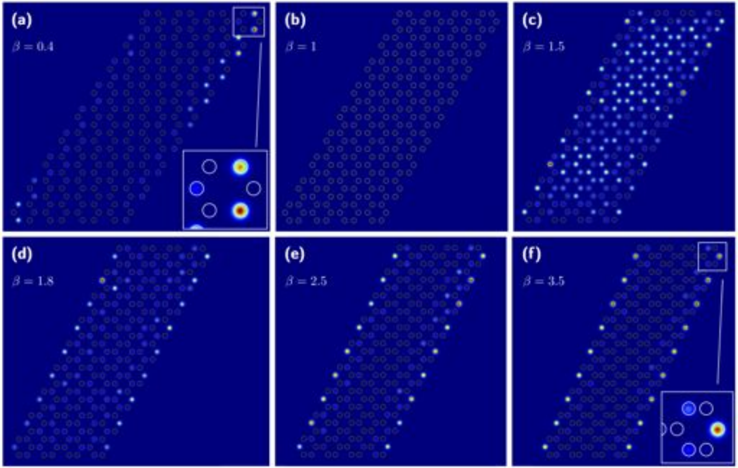


Diagram of existence obtained via a tight-binding approach

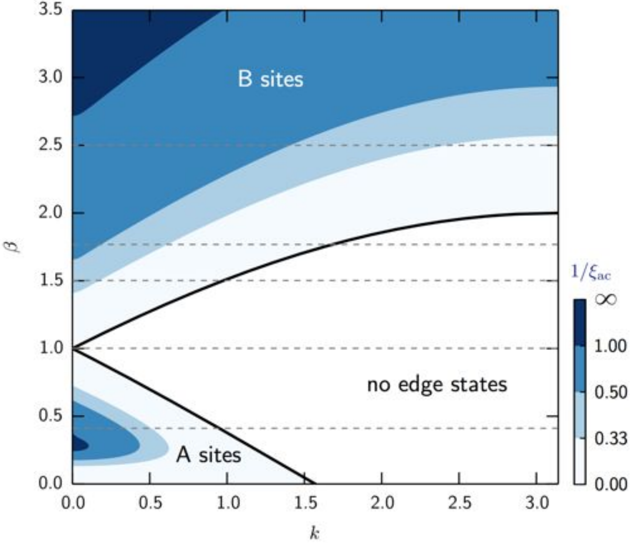


Observation of **armchair** edge states at Dirac frequency



- No edge states along anisotropy (horiz.) axis $\longrightarrow$ Zak phase
- States live only on one sublattice (A for $\beta < 1$ and B for $\beta > 1$)
- Edge states apparition doesn't depend on the phase transition
- The localisation length ξ decreases with $\beta > 1$

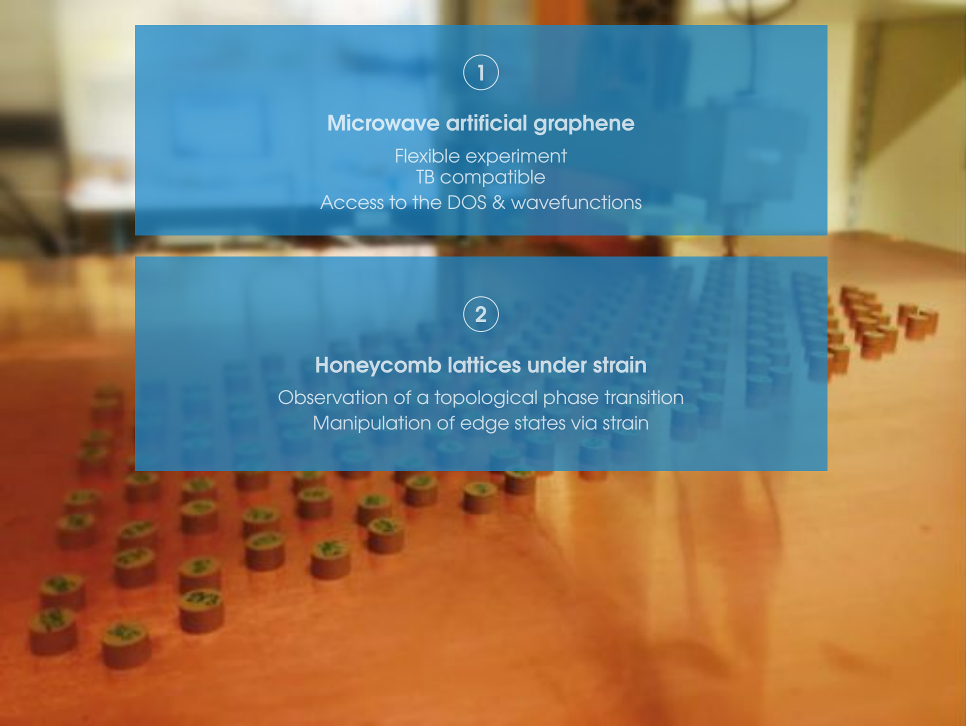
Diagram of existence obtained via a tight-binding approach



1

Microwave artificial graphene

Flexible experiment
TB compatible
Access to the DOS & wavefunctions



1

Microwave artificial graphene

Flexible experiment
TB compatible
Access to the DOS & wavefunctions

2

Honeycomb lattices under strain

Observation of a topological phase transition
Manipulation of edge states via strain

1

Microwave artificial graphene

Flexible experiment
TB compatible
Access to the DOS & wavefunctions

2

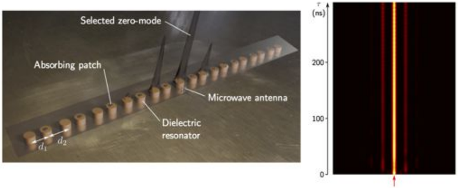
Honeycomb lattices under strain

Observation of a topological phase transition
Manipulation of edge states via strain

More info : Phys. Rev. Lett. **110**, 033902 (2013) – Phys. Rev. B **88**, 115437 (2013)
ArXiv : 0000.0000 (2014) available soon :-)

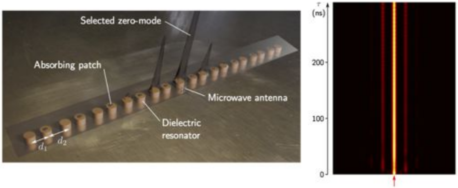
mail : bellec@unice.fr – **web** : www.unice.fr/mbellec

- Inhomogeneous strain
  $\rightarrow$ pseudo-magnetic field, Landau levels, *etc.*
- Quantum search algorithm (collab. Univ. Nottingham)
- Quasicrystals (collab. INLN)
- Selective enhancement of topologically induced interface states (collab. Univ. Lancaster)



Future work

- Inhomogeneous strain
  → pseudo-magnetic field, Landau levels, *etc.*
- Quantum search algorithm (collab. Univ. Nottingham)
- Quasicrystals (collab. INLN)
- Selective enhancement of topologically induced interface states (collab. Univ. Lancaster)



Thanks for your attention !