# FORMATION OF SATELLITES from massive rings in the Solar system 



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()bservatoire

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## Épiméthée

Dione Prométhée
Téthys



## JUPITER



## URANUS



## NEPTUNE



## ALL GIANT PLANETS



## ORIGIN of the SATELLITES ?

Planets form in a gas/dust disk around the Sun.

A giant planet carves a gap in the disk, and has its own circumplanetary disk.

A mini-planetary system would then form around the planet.
( Canup \& Ward 2002, 2006 ;
Sasaki et al. 2010 ; Mosqueira \& Estrada 2003a,b... )

But the satellites systems don't look like the Solar System.


Distributions of giant planets' regular satellites:
> don't reach the planet
> ranked by mass
> pile-up at a few planetary radii (small bodies)

Why?


It's not a power law, which question the Circum-Planetary Disc model...

## CONCLUSION



## TECHNICAL INTRODUCTION

1) Kepler's Law
2) Roche's radius

## Kepler's Law


« The cube of the radius of an orbit is proportionnal to the square of the period.»
planet:
period P [year] radius R [AU]
$\mathrm{P}^{2} / \mathrm{R}^{3}$

Mer Earth Jup
$0,25 \quad 1 \quad 11,8$
$0,4 \quad 1 \quad 5,2$
$1 \quad 1$
11

OR : The closer from the Sun (or Saturn), the faster the rotation.

## Kepler's Law : Application

Take 2 bodies in orbit around Saturn (within the rings or outside).

The inner one rotates faster than the outer one.

But they attract each other, which accelerates the outer one, and slows down the inner one.

## Kepler's Law : Application



## Spreading of the rings




We found that independantly of the initial mass, the rings should have their present mass after 4.5 Gyrs. (Salmon, Charnoz, Crida, Brahic 2010)

The rings could have been much more massive in the past.

## TECHNICAL INTRODUCTION

## 1) Kepler's Law

2) Roche's radius

## TIDES



Black arrows: Gravitational force due to Moon. White arrows: Net differential force relative to centre of the Earth - the tide-raising force.

## TIDES

Reminder: Tidal forces (per mass unit) :

$$
\begin{aligned}
& \Omega=\left(G M / r^{3}\right)^{1 / 2} \\
& \mathrm{~F}_{\mathrm{g}}=G M /(r+/-\mathrm{a})^{2} \\
& \mathrm{~F}_{\mathrm{c}}=\Omega^{2}(r+/-\mathrm{a}) \\
& \mathrm{F}_{\mathrm{t}}=3 \Omega^{2} \mathrm{a}
\end{aligned}
$$



## Roche Radius

Self-gravity force of the two spheres (per mass unit) :
$F_{\mathrm{sg}}=G^{\star}(4 / 3) \pi \rho a^{3} /(2 a)^{2}$

Condition for stability of the aggregate: $\mathrm{F}_{\mathrm{sq}}>\mathrm{F}_{\mathrm{t}}$, or : $\quad r>(9 M / \pi \rho)^{1 / 3}=r_{\text {Roche }}$

Application:
$M=M_{\text {saturn }}$,
$\rho=600 \mathrm{~kg} \cdot \mathrm{~m}^{-3}$
$r_{\text {Roche }}=1,410^{8} \mathrm{~m}$

$\Omega$

## Application :

Saturn's rings extend to 136000 km.

## Composition:

$10 \mathrm{~cm}-10 \mathrm{~m}$ blocks, of $90 \%$ water ice!
$\rightarrow$ Tidal disk.

Movie: Hanno REIN.

## TECHNICAL INTRODUCTION SUMMARY:

1) The rings can't agglomerate because they are inside the Roche radius.
2) But they spread...
(and were much more massive in the past)
3) Satellites migrate outwards, repeled by the rings.
In particular, Janus should have been inside the rings 100 Myrs ago !

## Satellites children of the rings 1) Saturn's small moons

Just outside Saturn's rings, there is a handfull of small satellites, with surprising properties:

- underdense (~600 kg.m-3)
- same spectrum as the rings
- dynamically young
- young surfaces



# Satellites children of the rings 1) Saturn's small moons 

But... we have seen that the rings spread...
On the inside, the ice falls into Saturn.
And on the outside?

After crossing the Roche limit, the ice boulders agglomerate, accrete, coallesce, and form new small satellites !

# Satellites children of the rings 1) Saturn's small moons 



# Satellites children of the rings 1) Saturn's small moons 

We made numerical simulations of this process, with viscous spreading of the rings, and satellite formation beyond $r_{\text {Roche }}$ (Charnoz, Salmon, Crida, 2010).

It works !
In ~100 Myrs, reproduction of the 5 smallest moons from rings like today's.

## Satellites children of the rings 1) Saturn's small moons

## Satellites children of the rings 2) A general model

Adding Saturn's tidal dissipation, we could also reproduce numerically the mid-sized moons of Saturn, explaining their composition
(Charnoz, Crida, et al. 2011).

## Satellites children of the rings 2) A general model

Now, let's put this simple model in equations !

## Spreading of a tidal disk

Inside the Roche radius $r_{R}$, there is a «tidal disk», that spreads with a mass flow $F$ (assumed constant).

Beyond $r_{R}$, new satellitte(s) form...


## Notations

1D model.

Be $T_{R}$ the orbital period at $r_{R}$, and
$\tau_{\text {disk }}=M_{\text {disk }} / \mathrm{FT}_{\mathrm{R}}$, the normalized life-time of the disk.

The disk spreads with a viscous time $t_{v}=r_{R}{ }^{2} / v$.
Using Daisaka et al. (2001)'s prescription for $v$, we find: $\tau_{\text {disk }}=t_{v} / T_{R}=0.0425 D^{-2} \quad$ where $D=M_{\text {disk }} / M_{p}$.

## Continuous regime

Say 1 satellite forms. Its mass is: $\quad M=F t$

It feels a torque from the tidal disk : $\Gamma=\frac{8}{27}\left(\frac{M}{M_{p}}\right)^{2} \Sigma r^{4} \Omega^{2} \Delta^{-3}$ where $\Delta=\left(r-r_{R}\right) / r_{R}$ (Lin \& Papaloizou 1979).
$\rightarrow$ Migration rate :

$$
\frac{d \Delta}{d t}=\frac{32}{27} q D T_{R}^{-1} \Delta^{-3}
$$

where $q=M / M_{p}$.

$$
\begin{equation*}
q=\left(\frac{\sqrt{3}}{2}\right)^{3} \tau_{\text {disk }}^{-1 / 2} \Delta^{2} \tag{3}
\end{equation*}
$$

We call this the continuous regime .

## Continuous regime

This holds as long as the satellite captures immediately what comes through $r_{R}$.

That is, as long as $\left(r-r_{R}\right)<2 r_{\text {Hill }}$, or $\Delta<2(q / 3)^{1 / 3}$.


Input into Eq.(3), this gives a condition of validity for the continuous regime :

$$
\begin{aligned}
& \Delta<\Delta_{c}=\sqrt{\frac{3}{\tau_{\text {disk }}}}=\sim 8.4 \mathrm{D} \\
& q<q_{c}=\frac{3^{5 / 2}}{2^{3}} \tau_{\text {disk }}^{-3 / 2}=\sim 222 \mathrm{D}^{3}
\end{aligned}
$$

Duration of the continuous regime: $10 \mathrm{~T}_{\mathrm{R}}$.

## Applications

Using the relation between $\tau_{\text {disk }}$ and $D$, one finds :

1) Earth's Moon forming disk :
$\mathrm{q}_{\mathrm{c}}=\sim$ mass of the Moon!
=> Agreement with N -body numerical simulations.
2) Charon never left the continuous regime.
3) Saturn's rings now :

$$
q_{c}=\sim 10^{-18} .
$$

=> Only very small objects are formed.

## Pyramidal regime

Satellites of mass $\mathrm{q}_{\mathrm{c}}$ are produced at $\Delta_{\mathrm{c}}$ every $\mathrm{q}_{\mathrm{c}} / \mathrm{F}$.
Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other.

If their distance decreases below 2 mutual Hill radii, they merge.

This leads to the formation of satellites of masses $2 q_{c}$, every $2 q_{c} / F$. They migrate away and merge further...

And so on, hierachicaly...
We call this the pyramidal regime.

## Pyramidal regime

- Using Eq.(2), we show that in the pyramidal regime, while the mass is doubled, $\Delta$ is multiplied by $2^{5 / 9}$.

Thus, $q \alpha \Delta^{9 / 5}$.
In addition, the number density of satellites should be proportionnal to $1 / \Delta$, explaining the pile-up.

- Beyond the 2:1 Lindblad resonance with $r_{R}(\Delta=0.58)$,

Eq.(2) doesn't apply. Migration is driven by planetary tides:

$$
\begin{equation*}
\frac{d r}{d t}=\frac{3 k_{2 \mathrm{p}} M \sqrt{G} R_{p}^{5}}{Q_{p} \sqrt{M_{p}} r^{11 / 2}} \tag{4}
\end{equation*}
$$

## Pyramidal regime

The result spectacularly matches the distribution of the Saturnian system!


## Pyramidal regime

The result spectacularly matches the distribution of the Saturnian, Uranian, and Neptunian systems !



## Summary

## 1) Continuous regime:

1 moon grows
$\mathrm{q} \alpha \Delta^{2}$ until $\Delta_{c}$ or $q_{c}$.
2) Discrete regime: transition, 2 moons
3) Pypannidal reaime:

Many moons
$\mathrm{q} \alpha \Delta^{9 / 5}$ or $\mathrm{r}^{3.8}$.

## Pyramidal regime (many moons)

# Continuous regime <br> (1 moon) 

$10^{\circ}$
$\begin{array}{cccc}0.0001 & 0.0010 & 0.0100 & 0.1000 \\ & & \begin{array}{c}\Delta \\ \text { (Normalised distance of first satellite to disk's edge) }\end{array}\end{array}$

## Summary

## Application :

Take $\mathrm{M}_{\text {disk }}=1.5 \mathrm{x}$ total mass of present satellite system.

Giant planets : dominated by the pyramidal regime,

## Earth and Pluto :

 1 large satellite.

## Conclusion \& Discussion

The spreading of a tidal disk beyond the Roche radius

- explains the mass-distance distribution of the regular satellites of the giant planets (observational signature of this process)
- unifies terrestrial and giant planets in the same paradigm.
$\bullet$ most Solar System regular satellites formed this way.
* Jupiter doesn't fit in this picture : probably formed in a
circum-planetary disk (Çanúp \& Ward 2002, 2006; Mosqueira \& Estrada 2003a,b)
- NB: Titan fits very well in this picture, though its «tidal age» is too large... Coincidence?


## Thanks!

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## ORIGIN of the SATELLITES ?

As for our Moon, the Earth didn't have a Circum-Planetary Disk, but a giant impact with a Mars-sized body ejected enough material (and angular momentum) around the Earth to form the Moon
( Canup \& Asphaug 2001 )


