Hybrid dimensional two phase flows in fractured porous media

Roland Masson
in collaboration with K. Brenner, M. Groza, J. Hennicker, L. Jeannin, G. Lebeau, S. Lopez, P. Samier, F. Xing

Laboratoire J.A. Dieudonné, Université Nice Sophia Antipolis, and team COFFEE, INRIA Sophia Antipolis, France

Numerical Methods for PDEs 2016
Workshop Industry and Mathematics
November 21st-23th, IHP, Paris
Outline

- Modelling Darcy flows in Discrete Fracture Networks
- The Vertex Approximate Gradient Discretization (VAG)
- Applications and Numerical Results
Fractured porous media: multiple scales (figures from J. R. de Dreuzy, Geosciences Rennes and team INRIA Sage)
Fractured porous media: applications

- Oil and gas exploration and production
- Hydrogeology
- Geothermal energy
- Geological storages
- Soil remediation
- ...
Flow in Fractured porous media: two main approaches

- **Double Continuum Media**: 3D fracture medium coupled to 3D matrix medium
- **Discrete fracture models (DFM)**: 2D fracture model coupled to 3D matrix medium

- Possibility to couple both approaches
  - Double Continuum media for small fractures coupled with DFM for large fractures
  - Numerical Homogeneization: parameters of the Double Continuum media computed by a DFM model
Modelling Darcy flows in Discrete Fracture Networks
Equi-dimensional model in phase pressures formulation

\[ u^\alpha : \text{phase pressure} \]
\[ \alpha = 1 : \text{wetting phase}, \quad \alpha = 2 : \text{non wetting phase} \]

\[ p = u^2 - u^1 : \text{capillary pressure} \]
\[ S^2(x, p) : \text{inverse of capillary pressure graph}, \quad S^1(x, p) = 1 - S^2(x, p) \]
\[ k^\alpha(x, S^\alpha) = \frac{k_r^\alpha(x, S^\alpha)}{\mu^\alpha} : \text{phase mobility} \]
Equi-dimensional model in phase pressures formulation

- Rock properties:
  - $\phi(x)$: porosity
  - $\Lambda(x)$: absolute permeability

- Fluid model:
  - Incompressible flow (fixed densities $\rho^\alpha$, $\alpha = 1, 2,$)
  - Immiscible flow

Darcy velocities:

$$ q^\alpha = -k^\alpha(x, S^\alpha(x, p)) \Lambda(x)(\nabla u^\alpha - \rho^\alpha g), \quad \alpha = 1, 2, $$

Volume conservation for each phase:

$$ \phi(x) \partial_t S^\alpha(x, p) + \text{div}(q^\alpha) = 0, \quad \alpha = 1, 2. $$
Equi-dimensional model: matrix and fracture domains

- Fracture: can act as drain or barrier
- Fracture width: $d_f \ll$ matrix size $L$
- Fracture rocktype (f): $\Lambda_f, \phi_f, k_f^\alpha, S_f^\alpha$
- Matrix rocktype (m): $\Lambda_m, \phi_m, k_m^\alpha, S_m^\alpha$
Dimensional Hybridizing (codimension 1 in the fracture)


- **Dimensional hybridizing**: averaging the model equations over the fracture width
- **Objectives**: facilitate the mesh generation and lower the number of degrees of freedom

**equi-dimensional model:**

- $(u, q)$
- $d/2$
- $n^+$
- $n^-$
- $n$

**hybrid-dimensional model:**

- $(u_m, q_m)$
- $j_b, j_n$
- $n^+$
- $n^-$
- $n$
Hybrid dimensional models

Matrix Darcy Law: \( \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_m)) \Lambda_m(\nabla u_m^\alpha - \rho^\alpha \mathbf{g}) \)

Matrix Vol. Cons.: \( \phi_m \partial_t S_m^\alpha(p_m) + \text{div}(\mathbf{q}_m^\alpha) = 0 \)

Fracture Darcy Law: \( \mathbf{q}_f^\alpha = -d_f k_f^\alpha(S_f^\alpha(p_f)) \Lambda_f,\tau(\nabla_\tau u_f^\alpha - \rho^\alpha \mathbf{g}_\tau) \)

Fracture Vol. Cons.: \( \phi_f d_f \partial_t S_f^\alpha(p_f) + \text{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_n^+ \mathbf{q}_m^\alpha + \gamma_n^- \mathbf{q}_m^\alpha = 0 \)
Transmission conditions at the matrix fracture interface

- Discontinuous pressure model:
  \[ \gamma_{n \pm} q_{m}^{\alpha} = q_{f,n \pm}^{\alpha} \approx \]
  \[ k_{f}^{\alpha} (S_{f}^{\alpha} (\gamma \pm p_{m})) \Lambda_{f,n} \left( \frac{\gamma \pm u_{m}^{\alpha} - u_{f}^{\alpha}}{d_{f}} - \rho^{\alpha} g \cdot n^{\pm} \right)^{+} \]
  \[ + k_{f}^{\alpha} (S_{f}^{\alpha} (p_{f})) \Lambda_{f,n} \left( \frac{\gamma \pm u_{m}^{\alpha} - u_{f}^{\alpha}}{d_{f}} - \rho^{\alpha} g \cdot n^{\pm} \right)^{-} \]

- Continuous pressure model \((\frac{\Lambda_{f,n}}{d_{f}} \gg \frac{\Lambda_{m,n}}{L})\):
  \[ \gamma_{+} u_{m}^{\alpha} = \gamma_{-} u_{m}^{\alpha} = u_{f}^{\alpha}. \]
Generalization to complex Discrete Fracture Network

- Pressure continuity and flux conservation is assumed at fracture intersections
- Zero flux is assumed at immersed fracture tips
The Vertex Approximate Gradient Discretization (VAG)
Discretization: state of the art

- MFE or MHFE: Jaffré et al 2002, Firoozabadi 2008
- XFEM type methods: Formaggia, Scotti et al 2012
- ...

Our contributions:

- For both continuous and discontinuous of models
  - Extension of the Gradient scheme framework (Eymard et al 2010)
  - VAG and HFV discretizations
  - Two phase flows
VAG scheme: degrees of Freedom (discontinuous pressure model)

- Two matrix cells touching a fracture face
  - Matrix d.o.f. (black)
  - Fracture d.o.f. (red)
VAG Matrix-Matrix and Fracture-Fracture Fluxes

- **mm (ff) fluxes:**
  - upwind (w.r.t. the phase mobility)
  - MPFA
  - local stencil to each cell $K$
    (fracture face $\sigma$)

  \[
  F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^-
  \]

  \[
  f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_{K}^{\nu\nu'}(u_K - u_{\nu'} - \rho(x_K - x_{\nu'}) \cdot g)
  \]

- **mm (ff) transmissivities:**
  - conforming $P^1$ FE gradient on a tetrahedral (triangular) submesh
    ($\{e_\nu\}_\nu = P^1$ FE Basis Functions)

  \[
  T_{K}^{\nu\nu'} = \int_{K} \Lambda \nabla e_\nu \nabla e_{\nu'} \, dx
  \]
VAG Matrix-Fracture Fluxes

- **mf fluxes:**
  - upwind (w.r.t. the phase mobility)
  - TPFA
  - Saturation jump at mf interfaces
  - Gravity in normal direction

\[
F_{\nu_m\nu_f}(u) = k_f(S_f(p_{\nu_m}))f_{\nu_m\nu_f}(u)^+ + k_f(S_f(p_{\nu_f}))f_{\nu_m\nu_f}(u)^-
\]

\[
f_{\nu_m\nu_f}(u) = T_{\nu_m\nu_f}(u_{\nu_m} - u_{\nu_f} - \frac{\rho d_f}{2} \mathbf{g} \cdot \mathbf{n}_{\nu_m\nu_f})
\]

- **mf transmissivities:**
  - Mass Lumping of $P^1$ FE basis function traces
Hybrid Continuous Pressure and Hybrid Discontinuous Pressure degrees of freedom

- Hybrid Discontinuous Pressure
- Hybrid Continuous Pressure
Hybrid Continuous Pressure model: Fluxes

- **Matrix mm fluxes**

  \[ F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^- \]

  \[ f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_{K}^{\nu\nu'}(u_K - u_{\nu'} - \rho(x_K - x_{\nu'}) \cdot \mathbf{g}) \]

- **Fracture ff fluxes**

  \[ F_{\sigma s}(u) = k_f(S_f(p_\sigma))f_{\sigma s}(u)^+ + k_f(S_f(p_s))f_{\sigma s}(u)^- \]

  \[ f_{\sigma s}(u) = \sum_{s' \in \partial \sigma} T_{\sigma}^{ss'}(u_\sigma - u_{s'} - \rho(x_\sigma - x_{s'}) \cdot \mathbf{g}) \]

- **No mf fluxes**
Hybrid Continuous Pressure model

Control Volumes

Saturations at matrix fracture interfaces

\[
\begin{align*}
S_{K,\nu}^\alpha &= S_{m}^\alpha(p_\nu), \\
S_K^\alpha &= S_{m}^\alpha(p_K), \\
S_{\sigma,s}^\alpha &= S_{f}^\alpha(p_s), \\
S_{\sigma}^\alpha &= S_{f}^\alpha(p_\sigma).
\end{align*}
\]
Applications and Numerical Results
Comparison of equi- and hybrid-dimensional models: with J. Hennicker, K. Brenner and P. Samier

- $\Omega = (0, 400) \times (0, 800) \text{ m}$
- Equi-dimensional mesh: 22500 triangles
- Hybrid dimensional mesh: 16900 triangles
- **Matrix:**
  \[
  \phi_m = 0.2, \quad \Lambda_m \text{ isotropic}
  \]
- **Faults:**
  \[
  d_f = 4m, \quad \phi_f = 0.4, \quad \Lambda_f \text{ isotropic}
  \]
- Injection of oil in the bottom fault
- Initially saturated with water
Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(S_m^0) = p_{c,f}(S_f^0) = 0$

Zero Capillary Pressure: $p_{c,m}(S_m^0) = p_{c,f}(S_f^0) = 0$

- Equi dim

- Hybrid Disc.

- Hybrid Cont.
Drains: $\Lambda_f / \Lambda_m = 1000; \ p_{c,m} \neq 0; \ p_{c,f} = 0$

Capillary Pressure: $p_{c,m}(S_m^o) = -10^5 \ln(S_m^o); \ p_{c,f} = 0$

- Equi dim
- Hybrid Disc.
- Hybrid Cont.
Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} = -10^5 \ln(S_m^o)$, $p_{c,f} = -10^4 \ln(S_m^o)$

- Equi dim

- Hybrid Disc.

- Hybrid Cont.
Drains: $\Lambda_f / \Lambda_m = 100; \quad p_{c,m} = -10^5 \ln(S_m^o)$

Equi-Dimensional Model: Saturation Stratification in the fault Network

\[ p_{c,f} = 0 \quad \quad p_{c,f} = -10^4 \ln(S_m^o) \]
Drain-Barrier: $\Lambda_f^{\text{drain}}/\Lambda_m = 1000; \Lambda_f^{\text{barrier}}/\Lambda_m = 0.01$

**Capillary Pressure:** $p_{c,m} = -10^5 \ln(S_m^o); p_{c,f} = 0$

- Equi dim
- Hybrid Disc.
Drain-Barrier: $\Lambda_f^{\text{drain}}/\Lambda_m = 1000; \Lambda_f^{\text{barrier}}/\Lambda_m = 0.01$

Equi dim.  

Hybrid Disc.
High energy geothermal test case: with F. Xing and S. Lopez (BRGM)

Matrix: \( T = 473 \text{ K} \), zero liquid flux

Fracture: \( T = 623 \text{ K} \), liquid flux

\( \Lambda_m = 10 \, mD \)
\( \Lambda_f = 1 \, D \)

\( \phi_m = 0.25 \)
\( \phi_f = 0.35 \)

\( d_f = 1 \, m \)

3000 m
Thermal two phase flow

**Model:** $H_2O$ component in liquid or vapor phase

**Unknowns:** $P, T, S^l, S^g$

**Continuous Pressure model**

$P, T, S$ formulation.

Conservation of mass, energy and volume:

\[
\begin{align*}
\phi \frac{\partial}{\partial t} \left( \sum_{\alpha=l,g} \rho^\alpha S^\alpha \right) + \text{div} \left( \sum_{\alpha=l,g} \rho^\alpha q^\alpha \right) &= 0, \\
\phi \frac{\partial}{\partial t} \left( \sum_{\alpha=l,g} \rho^\alpha e^\alpha S^\alpha \right) + (1 - \phi) \frac{\partial}{\partial t} E_r + \text{div} \left( \sum_{\alpha=l,g} \rho^\alpha h^\alpha q^\alpha - \lambda \nabla T \right) &= 0, \\
S^l + S^g &= 0,
\end{align*}
\]

+ liquid vapor thermodynamical equilibrium:

\[
\begin{align*}
T &= T_{sat}(P) \text{ if } S^g > 0 \text{ and } S^l > 0, \\
S^g &= 0 \text{ if } T < T_{sat}(P), \\
S^l &= 0 \text{ if } T > T_{sat}(P).
\end{align*}
\]
High energy geothermy: temperature and gas saturation
High energy geothermy: temperature and gas saturation

\[ S_m^g \text{ and } T_f \]

\[ T_f \text{ and } T_m > 500 \text{ K} \]
Two phase hybrid dimensional model taking into account
- networks of fractures
- drains and barriers
- discontinuous capillary pressures
- gravity

On going work
- Convergence analysis using the gradient scheme framework (with J. Droniou)
- Geothermal systems: ANR project with BRGM, Storengy, MdS, LJLL, LJAD
- Large networks
Acknowledgements

Thanks for your attention and to Total, Engie and BRGM for their support.