Report1: Anemos Task 3.3.4

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Introduction

- Representing the complex magnetic and material geometry.
  For the simulation of ELMs and control by pellets in ITER.
- Geometric approximation by triangles lead to inaccuracy.
  B-splines reproduce the geometry of the domain exactly.
- In JOREK, isoparametric formulation uses Bezier patches in 2D.
  A gain up to an order of magnitude is expected.
  Meshing need to be improved for ITER tokamak.
- $\rightarrow$ We propose a class of parameterizations to make a separate mesh generation unnecessary.

Tokamak Shape

- The idea here is to rely on the geometric rigidity of the union of isobaric curves, and approximate by algebraic data.
- The shape of the plasma boundary is identifiable.
- Usual control of the plasma shape during a plasma discharge, rely on coils current, with a feedback loop.
- The required shape is maintained, in a stationary manner, in order to avoid sudden termination of the plasma (when the plasma touches the first wall).
- But with this procedure, it is difficult to compute the internal magnetic flux configuration.

X shaped curve

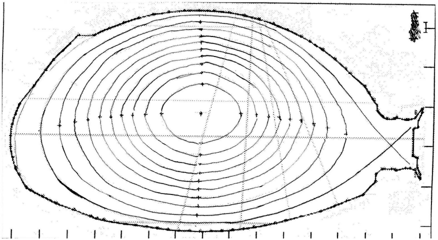

Figure 1: Isobaric curves

Algebraic model

- One observes a "topological rigidity" of the picture of all the isobaric curves, illustrated in previous Figure 1.
- The idea is to approximate it by algebraic data which are more "rigid" than meshes.
- We propose a special type of parameterizations by tensor product B-splines:
  A kind of PHT over what we call a S-mesh.

Steps

1. Fix a rough (radial) quadrangular mesh. In each quadrangle, Construct the piecewise tensor product $B(3,3)$-function approximating the solution of the PDE.
2. Detect the cell containing the X point. Then approximate the corresponding level set.
3. Decompose the domain into curved quadrangle lined with level sets.
4. Parameterize these quadrangles.
5. Follow, via linearization, the deformed parameterization corresponding to the new solution for $t + \Delta t$.

Level sets

- The curves in Figure 1 are isobaric curves.
- The important feature is that one of them has a node (X point).
- The curves of Figure 1 roughly resemble the level sets of a parameterized curve shown in next Figure 2.
- $\rightarrow$ We first experiment with these level sets, which are easy to compute.

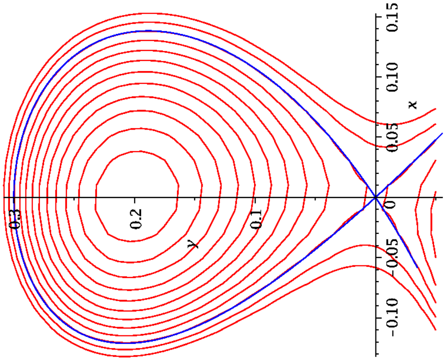

Figure 2: Parameterized curve

Segment of curve

- We approximate a (small) segment of curve $F = cst$ by a 3-Bezier.
- We assume that the two extremal points A and B are given, with the first and second derivatives of F.
- We compute the two tangents and their intersection C.
- So the 4 points of the control polygon are A, $C1$, $C2$ and B. With $AC1 := c1 * AC$ and $BC2 := c2 * BC$.
- From the signs of the curvatures we choose a model without or with inflection.
  Then we estimate $c1$ and $c2$.

Bcurves

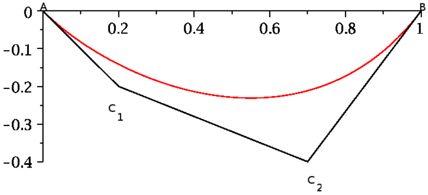

Figure 3: 3Bezier without inflection

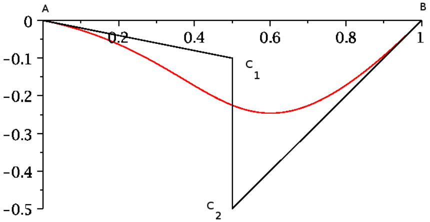

Figure 4: 3Bezier with inflection

Small surface

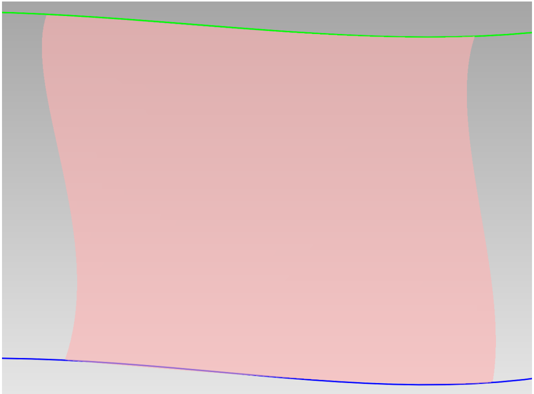

Figure 5: A surface and its borders

Parameterization of small surfaces

- Given a (small) surface delimited by level sets and 4 points A, B, A' and B'.
- We assume that we also have, as above, the 4 control polygons.
- Then we can construct a bicubic plane (small) surface with these borders. (Coons optimized)
- $\rightarrow$ A family of parameterized curves which interpolate the input (border) ones. $\rightarrow$ This defines refined quadrangular meshes.

Parameterization of a small surface

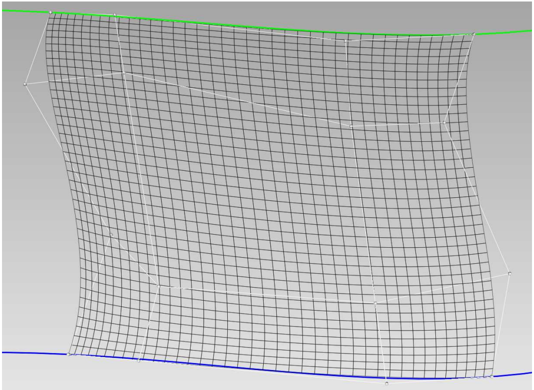

Figure 6: A parameterized surface and its borders

Injectivity and Transverse cones

- For refinements, we need that the curves of the parameterization do not intersect each other.
- In a previous work, we studied the following Injectivity Criterion.
- It uses the vectors defined by successive control points in each direction of the control net of (x, y).
- The two cones spanned by the two families of vectors should be disjoint.
  See Figure 7.

Criterion

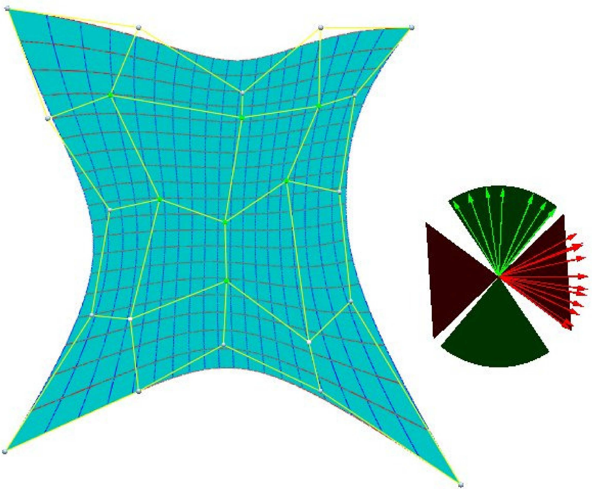

Figure 7: Transverse cones

Global view

- With the X geometry of the tokamak, we decompose the global surface.
- We consider a decomposition into n, e.g. $n = 8$, curved rectangles. We can subdivide the decomposition and increase n. See below.
- We then get parameterizations, locally similar to the ones above.
- $\rightarrow$ Meshes which approximate the isobar curves.

Decomposition in the physical domain

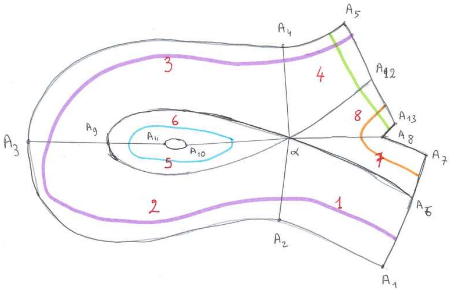

Figure 8: 8 curved rectangles

Parametric mesh

- To get a global parameterization C^1, we need to generalize the notion of Bsplines over a T-mesh. We call it S-splines.
- We allow an equivalence relation between points and also between edges.
- The following Figure, made of two rows, organizes 8 squares.
  The equivalence classes of points are 2 pairs of points and also a set of 4 points. The equivalence classes of edges are 4 pairs of edges. Coherently.
- $\rightarrow$ They can be refined and will map to the physical domain.

a S-mesh over the plan

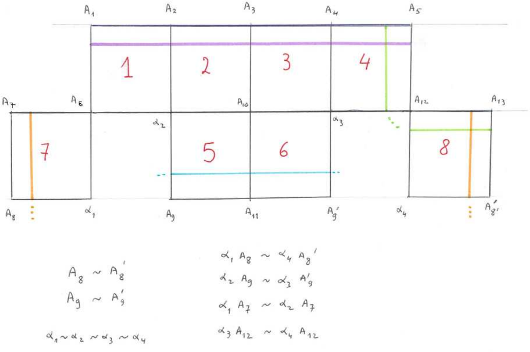

Figure 9: A S-mesh with $n = 8$ squares

Subdivision of the previous S-mesh

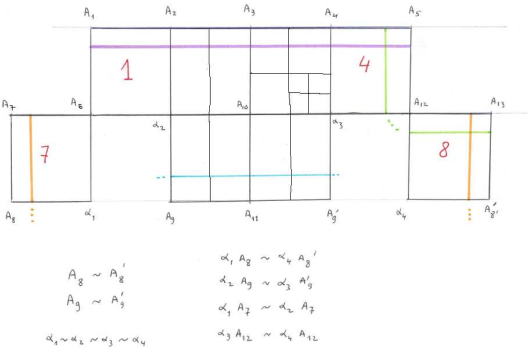

Figure 10: A S-mesh with $n = 17$ squares

Dimension formula

Proposition: The $(3,3)-$splines spaces over the previous type of S-mesh is a linear vector space E. Its dimension is equal to 4 times the number of equivalent classes of vertices.

Example: For the previous S-mesh, with $n = 8$, the dimension of E is $4 * 14 = 56$.

In the 2 next slides we present the graphs of 2 function of E.

Note that they take the same values on equivalent edges (resp. vertices).

a base function C^1

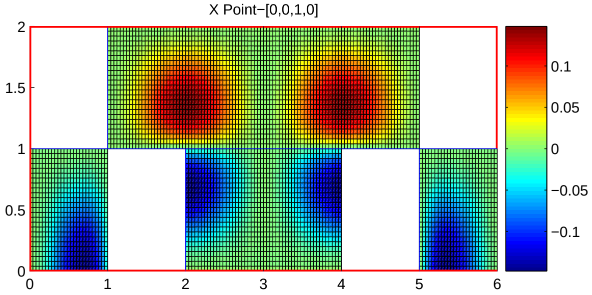

Figure 11: Values by colors over a S-mesh

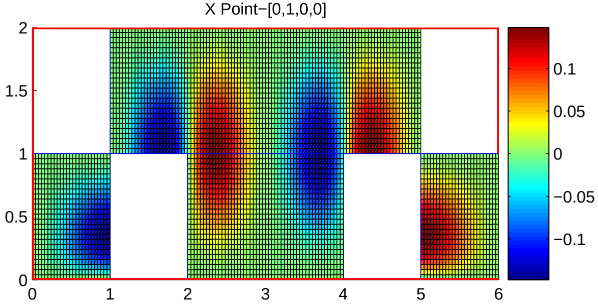

Figure 12: Values by colors over a S-mesh

Interpolation-Projection

- Given a set of conditions of values or tangency on the physical plane surface, we want to find a S-spline function which either satisfies or approximate these conditions.
- Since these conditions are linear, we end up solving a linear system.
- If there are more conditions than the dimension of the spline space E, we rely on usual techniques of approximate linear algebra.
- Of course, all the art will be to choose "well tuned" conditions.

Simple example

- We considered the simple case of a cubic curve given by its implicit equation
$y^2 - x(x-1)^2$.

- With the previous S-mesh, $(n = 18)$ we considered the corresponding conditions, in order to interpolate two functions x and y defining an image of the S-mesh.

Interpolation

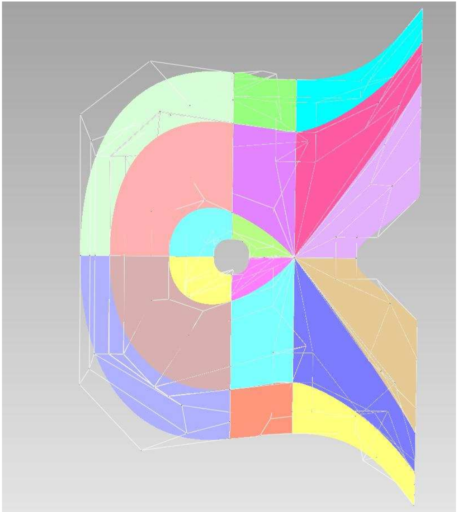

Figure 13: An interpolation with 18 squares

Parameterization

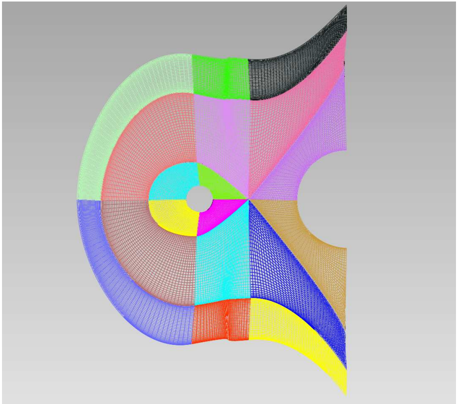

Figure 14: A parameterization with 18 squares

Conclusion, future developments

• We have presented a new model of splines that we called S-splines. It is adapted to represent isobaric curves in a Tokamak such that one curve has a X-point.

• It resembles the techniques used in Jorek for defining isoparametric finite elements.

• We made a good mathematical start and experimented with a simple algebraic model.

→ Now, we need to tune it and apply it to data corresponding to a Tokamak.