Colloque "Singularités Réelles et Systèmes Dynamiques"

Nice 16-19 Mai 2011

Fabrizio Broglia (Pise). On Pfister's multiplicative formulae for the ring of real analytic functions.

The classical Pfister's formula concerns the representation of the product of two elements of a field K wich are sums of 2^r squares. We generalize this fact in the sense that we prove that "sheaf product" of infinitely many sums of squares is again a sums of squares and we give some consequences related to 17th Hilbert problem. (joint work with Francesca Acquistapace and Jose F.Fernando)

Edward Bierstone (Toronto). Birational models with normal crossings singularities.

A survey of recent results on the question of resolving singularities except for normal crossings. What singularities persist as limits of normal crossings? Normal forms of limiting singularities – relationship with the desingularization invariant and Abhyankar-Jung phenomena.

Georges Comte (Chambéry). Rotation number of trajectories of Lipschitz vector fields. This talk concerns a joint work with Yosef Yomdin. Given a vector field in some open subset of R^3 we define the rotation number of two of its trajectories in the same way as for closed trajectories, that is to say for links, using an integral formula. We then prove that this rotation number is bounded from above, in the time interval T, by $A+BKT^2$, where A and B are universal coefficients and K is the Lipschitz constant of our vector field. As a by-product we obtain a bound from above, in terms of Lipschitz constant of the field, for the so-called Arnol'd-Hopf asymptotic invariant of a divergence-free vector field in some compact domain of R^3.

Sorin Dumitrescu (Nice). Dynamique de l'algèbre de Killing d'une connexion analytique réelle.

On étudie la dynamique des champs de Killing locaux d'une connexion analytique réelle sur une surface compacte. Dans le cas où les champs de vecteurs sont de divergence nulle et l'action est transitive sur un ouvert non vide, on en déduit que l'action est transitive partout.

Olivier Le Gal (Valladolid). New instances of o-minimality for solution of 2 dimensional systems of ODE.

We consider a system of two differential equation $Y'(x) = G(x, Y(x)), x > 0$, and prove the o-minimality of its solution $Z(x)$ in the following situations :

a. The system comes from an analytic vector field of $\mathbb{R}^3$, Z is non-oscillating and belong to an interlaced integral pencil.
b. G is definable in an o-minimal structure, affine with respect to Y, and Z belongs to a non-interlaced integral pencil

Lubomir Gavrilov(Toulouse). On the number of limit cycles which appear by perturbation of two-saddle cycles of planar vector fields.

We find an upper bound to the maximal number of limit cycles, which bifurcate from a hamiltonian two-saddle loop of an analytic vector field, under an analytic deformation. The case of a quadratic vector field will be studied in detail.

Pavao Mardesic (Dijon). On Uniform Boundedness of the Number of Zeros of Pseudo-Abelian Integrals .

Abelian integrals are integrals $I(t) = \int_{\delta(t)} \omega$ of polynomial (or rational) 1-forms along cycles $\delta(t) \in H_1(P^{-1}(t))$, $P \in \mathbb{C}[x, y]$. Abelian integrals appear as principal part of the Poincaré displacement function of the deformation $dP + \epsilon\omega$ along $\delta(t)$. Their zeros correspond to limit cycles born in the deformation.

Varchenko and Khovanskii prove the existence of a bound, uniform with respect to the degree of P and ω, for the number of these zeros.

Arnold posed with insistence the analogous problem for more general polynomial deformations of integrable systems. In particular for deformations of systems having a Darboux first integral $P = \Pi P_i^{a_i}$, $P_i \in \mathbb{C}[x, y]$ (called also logarithmic foliations). Then instead of abelian integrals, one encounters pseudo-abelian integrals.

I will present some joint results obtained with M. Bobienski and D. Novikov in our program to generalize the theorem of Varchenko-Khovanskii to pseudo-abelian integrals.

Maria Michalska (Łódź, Chambéry). Stability of algebras of bounded polynomials in two variables.

Given a set $S_c = \{(x, y) \in \mathbb{R}^2 : f(x, y) \leq c\}$ we show that any polynomial which is bounded on S_c is bounded also on S_d as long as there is no real bifurcation value of the complexification of f between the real numbers c and d. We will discuss this result and point out some of its consequences.

Daniel Panazzolo (Mulhouse). Center manifolds for holomorphic three-dimensional vector fields.

According to the theorem of resolution of singularities for vector fields in three dimensions, any singularity can be reduced to a certain list of models called "canonical". We will discuss the problem of existence of invariant surfaces (called center manifold) defined in sectorial vicinity of such singularities (in collaboration with M. McQuillan).

Armin Rainer (Wien). The exponential law for quasianalytic Denjoy-Carleman classes.

We will show that for all quasianalytic (and non-quasianalytic) Denjoy-Carleman classes C^M with natural stability properties the exponential law $C^M(E \times F, G) \cong C^M(E, C^M(F, G))$ holds for admissible locally convex vector spaces E, F, G. This requires a calculus for Denjoy-Carleman classes beyond Banach spaces (even if $E = F = G = \mathbb{R}$!). We shall explain the main principles and give applications to perturbation theory and manifolds of mappings. (Joint with A. Kriegl and P. Michor)

Guillaume Rond (Marseille). Le theoreme d'Abhyankar-Jung .

On considere un polynome de Weierstrass a coefficients dans un anneau de series formelles sur un corps de caracteristique nulle. Le theoreme d'Abhyankar-Jung nous dit que si le discriminant de ce polynome est a croisement normaux (c'est-a-dire si ce polynome est quasi-ordinaire), alors les racines de ce polynome sont des series de Puiseux en plusieurs variables. Nous allons donner une nouvelle preuve de ce resultat. Pour cela nous allons montrer, qu'apres transformation de Tchirnhausen, le polyhedre de Newton d'un polynome quasi-ordinaire est contenu dans le polyhedre de Newton d'un polynome quasi-homogene. Nous en deduirons une version du theoreme d'Abhyankar-Jung pour des anneaux henseliens qui n'admettent pas necessairement de theoreme de preparation de Weierstrass ; ce qui est le cas, en particulier, des germes de fonctions quasi-analytiques.

Reinhard Schaefke (Strasbourg). Stokes Phenomena and Resurgence.

Let us call here "resurgent" a function analytic near the origin all of whose singularities for analytic continuation are branch points of polynomial growth and such that the growth along almost all infinite rays is at most exponential. This is a special case of the notion introduced by Jean Ecalle.

It is well known that the Laplace transform of such a resurgent function admits a particular Stokes phenomenon.

Unfortunately, it is often difficult to prove the resurgence of some function obtained as formal Borel transform of a series solution of a differential equation.

The purpose of the lecture is to present a proof of the resurgence of the formal Borel transform of a series if we can associate it with a certain algebra of functions admitting "coherent" Stokes phenomena.

Tamara Servi (Lisbone). Algebraic independence of Weierstrass p-functions and definability.

It is well known that the real exponential field is o-minimal (Wilkie) and that no restriction of the sine function is definable in this structure (Bianconi). In analogy with this result, we consider two Weierstrass p-functions f and g such that the associated elliptic curves are non-isogenous. We prove that no restriction of g is definable in the o-minimal expansion of the real field by the real and imaginary parts of f restricted to a bounded domain (joint work with Gareth Jones). In my talk I will review the basic properties of Weierstrass p-functions and outline the proof of the mentioned result, which combines techniques from number theory and from o-minimality.

Stanisław Spodzieja (Łódź). Separation of real algebraic sets and the Łojasiewicz exponent.

We give an effective estimations for the local and global Łojasiewicz exponent of arbitrary real polynomial. More precisely, for an analytic function $f : (\mathbb{R}^n, a) \to (\mathbb{R}, 0)$ there exists constants $C > 0$ and $\varrho \in (0,1)$ such that $|\nabla f(x)| \geq C|f(x)|^{\varrho}$ in a neighbourhood of the point a. Then we prove $|f(x)| \geq C' \operatorname{dist}(x, V)^{1/(1-\varrho)}$ in a neighbourhood of a for some $C' > 0$, where $V = \{x \in \mathbb{R}^n : f(x) = 0\}$. If additionally f is a polynomial function of degree d, then $|f(x)| \geq C' \operatorname{dist}(x, V)^{d(3d-3)^{n-1}}$ in a neighbourhood of the point a. We also prove the global Łojasiewicz inequality : if g, $h : \mathbb{R}^n \to \mathbb{R}^m$ are polynomial mappings, $X = \{x \in \mathbb{R}^n : g(x) = 0\}$, $Y = \{x \in \mathbb{R}^n : h(x) = 0\}$ and $d = \max\{\deg g, \deg h\}$, then for some positive constant C'',

$$\operatorname{dist}(x, X) + \operatorname{dist}(x, Y) \geq C'' \left(\frac{\operatorname{dist}(x, X \cap Y)}{1 + |x|^2} \right)^{d(6d-3)^{n-1}} \quad \text{for} \quad x \in \mathbb{R}^n,$$

in particular,

$$|g(x)| \geq C'' \left(\frac{\operatorname{dist}(x, X)}{1 + |x|^2} \right)^{d(6d-3)^{n-1}} \quad \text{for} \quad x \in \mathbb{R}^n.$$

Laurent Stolovich (Nice). Normal forms of analytic perturbations of quasihomogeneous vector fields. We study germs of holomorphic vector fields which are "higher order" perturbations of a quasihomogeneous vector field in a neighborhood of the origin of $\mathbb{C}^n$, fixed point of the vector fields. We define a "diophantine condition" on the quasihomogeneous initial part S which ensures that if such a perturbation of S is formally conjugate to S then it is also holomorphically conjugate to it. We also study the normal form problem relatively to S.