Reduction theorem and normal forms of linear second order mixed type PDE families in the plane

Alexey Davydov

Vladimir State University, Russia; IIASA, Laxenburg, Austria
davydov@vlsu.ru, davydov@iiasa.ac.at

For a second order partial differential equation in the plane

$$a(x,y)u_{xx} + 2b(x,y)u_{xy} + c(x,y)u_{yy} = F(x,y,u,u_x,u_y),$$

where x, y are coordinates, a, b, c are smooth functions, and F is some function, the *characteristic equation* is defined as

$$a(x,y)dy^2 - 2b(x,y)dxdy + c(x,y)dx^2 = 0.$$

Characteristic directions at a point are the solutions of this equation. In a generic case the complete list of local normal forms of this equation was obtained in the end of 20-th century, when smooth normal forms were found near a point of the type change line $b^2 - ac = 0$, at which the characteristic direction is tangent to the line [1], [2]. It was proved that the equation near a point of such tangency is reduced to the form

$$dy^2 + (kx^2 - y)dx^2 = 0$$

where k is some real parameter, by multiplication on smooth nonvanishing function and an appropriate selection of new smooth coordinates with the origin at this point, if some standard conditions take place.

We generalized some of this results for the case of finite parametric families of characteristic (or PDEs) equations [3].

References

[1] A.A. Davydov - *The normal form of a differential equation that is not solved with respect to derivative, in the neighbourhood of its singular point*// Functional Anal. Appl., vol. 19 (1985), 81-89.

[2] A.A. Davydov, E.Rosales-zález - *Complete classification of a generic seond order linear PDE on the plane*//DAN USSR, 1996. T. 350, No. 2, 151 -154.

[3] A.A. Davydov, L. Trinh Thi Diep-*Reduction theorem and normal forms of linear second order mixed type PDE families in the plane*//Russian Mathematical Survay, V. 65, No. 5. (2010).