

Séminaire d'Algèbre, Topologie et Géométrie

Jeudi 3 mai à 14h00

Salle I

Michael Falk

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Title : *Critical points of master functions.*

Abstract : We study the critical points of functions $\prod_{i=0}^n \alpha_i^{\lambda_i}$, where $\alpha_0, \dots, \alpha_n$ are homogeneous linear forms and $\lambda_0, \dots, \lambda_n$ are complex weights satisfying $\sum_{i=0}^n \lambda_i = 0$. These *master functions* and their critical points play the central role in the Bethe Ansatz, which produces eigenvectors for commuting hamiltonians in certain quantum integrable systems.

For generic weights the critical points of Φ are isolated and non-degenerate, and their number is determined by the combinatorics of the associated hyperplane arrangement. In this talk we will show how a linear syzygy among polynomial master functions gives rise to families of master functions with positive-dimensional critical sets. The existence of such syzygies can be characterized combinatorially in terms of projections of Bergman fans, geometrically in terms of linear systems of hypersurfaces, and topologically in terms of resonance varieties. Everything will be illustrated with examples, including some arising from the Bethe Ansatz and some new examples (discovered by undergraduate research students) involving arrangements arising from graphs.