Algebraic and Combinatorial Structures in Quantum Field Theory

at

INSTITUT D'ETUDES SCIENTIFIC DE CARGESE

March 23 - April 3, 2 0 0 9

Organizers
P. Cartier, F. Patras, J.-Y. Thibon, K. Ebrahimi-Fard

March 25, 2009

Participants

Lecturers, 1st week (23 – 27.3.)

- Christian BROUDER (LMCP - CNRS, Paris)
- Pierre CARTIER (IHÉS, Bures-sur-Yvette)
- Yuri DOKSHITZER (LPTHE - CNRS, Paris)
- Vincent RIVASSEAU (Univ. Paris Sud)
- Jean ZINN-JUSTIN (SPhT, Saclay)
- Jean-Bernard ZUBER (LPTHE, Paris)

Speakers, 2nd week (30.3. – 3.4.)

- Carlo ALBERT (Geneva)
- Marc BELLON (Paris)
- Frédéric CHAPOTON (Lyon)
- Bertrand DELAMOTTE (Paris)
- Bertrand DUPLANTIER (Saclay)
- Loïc FOISSY (Reims)
- Alessandra FRABETTI (Lyon)
- Jose M. GRACIA-BONDÍA (Zaragoza)
- Razvan GURAU (Waterloo)
- Stefan HOLLANDS (Cardiff)
- Kai Johannes KELLER (Hamburg)
- Thomas KRAJEWSKI (Marseille)
- Govind KRISHNASWAMI (Durham)
- Jean-Louis LODAY (Strasbourg)
- Dominique MANCHON (Clermont-Ferrand)
- Nikolay NIKOLOV (Sofia)
- Daniele ORITI (Potsdam)
- Walter VAN SUIJLEKOM (Nijmegen)
- Rainer VERCH (Leipzig)
- Fabien VIGNES-TOURNERET (Vienna)

Schedule for the 1st WEEK: School

Monday	Tuesday	Wednesday	Thursday	Friday
	9:00-10:30 **Cartier I**	9:00-10:30 **Cartier II**	9:00-10:30 **Brouder II**	9:00-10:30 **Rivasseau II**
	coffee	*coffee*	*coffee*	*coffee*
arrival **lunch 13:00**	10:45-12:15 **Zuber I**	10:45-12:15 **Zuber III**	10:45-12:15 **Rivasseau I**	10:45-**12:15** **Dokshitzer II**
15:30-17:00 **Brouder I**	12:15-15:45 **lunch**	**lunch**	12:15-15:15 **lunch**	**lunch** **departure**
break	*coffee*	free	*coffee*	
17:15-19:15 **Zinn-Justin I**	16:00-18:00 **Zinn-Justin II**		15:30-18:00 **Dokshitzer I**	
	break		*break*	
	18:15-19:15 **Zuber II**		18:15-19:45 roundtable session **Flume**	

Schedule for the 2nd WEEK: Conference

Monday	Tuesday	Wednesday	Thursday	Friday
	9:00-10:00 **BELLON**	9:00-10:00 **VERCH**	9:00-10:00 **CHAPOTON**	9:00-10:00 **GRACIA-BON**
	coffee	*coffee*	*coffee*	*coffee*
	10:15-11:15 **FOISSY**	10:15-11:15 **VAN SUIJLE**	10:15-11:15 **LODAY**	10:15-11:15 **HOLLANDS**
	11:15-12:15 **KRISHNAS**	11:15-12:15 **ALBERT**	11:15-12:15 **FRABETTI**	11:15-**12:15** **DELAMOTTE**
arrival **lunch 13:00**	**lunch**	**lunch**	**lunch**	**departure**
15:30-16:30 **KRAJEWSKI**	15:30-16:30 **NIKOLOV**		15:30-16:30 **MANCHON**	
coffee	*coffee*	free	*coffee*	
16:45-17:45 **DUPLANTIER**	17:00-18:00 **KELLER**		16:45-17:45 **ORITI**	
18:00-19:00 **VIGNES-T**			18:00-19:00 **GURAU**	

Titles + Abstracts: School

- Ch. Brouder: I+II, 3hrs

Hopf algebraic structure of quantum field and many-body theories

Three lectures describe some of the Hopf algebraic structures that are present in perturbative quantum field theory and many-body physics.

The first lecture introduces, at a very elementary level, the basic concepts of Hopf algebras. Examples show that Hopf algebras are already present in basic calculus and quantum mechanics. The Hopf algebra of quantum fields is described. Wick's theorem is conveniently described by the Hopf algebraic concept of twisted product.

The second lecture uses the algebraic notions introduced in the first lecture to describe the basic combinatorics of Feynman diagrams. By purely algebraic means, the diagrammatic expansion is recovered with its combinatorial coefficients. The power and simplicity of the Hopf algebraic tools are demonstrated by giving detailed derivations valid in any order of perturbation theory. This approach is applied to the calculation of Green functions and extended to systems where the initial state is a general state instead of the vacuum.

The third lecture is devoted to the structure of Green functions and of renormalization. For that purpose, a second coproduct is introduced and the relation between this second coproduct and the Hopf algebraic coproduct turns out to be the key to the structure of Green functions. Bogoliubov's concept of generalized vertex is well described by this approach, that is applied to the derivation of renormalization identities.

- P. Cartier: I+II, 3hrs

Hopf Algebras, Theory and Applications to QFT

We intend to give an introduction to the mathematical theory of Hopf algebras and some of their applications to problems of Quantum Field Theory .

We begin by introducing two Hopf algebas associated to Lie groups. The first one is the convolution algebra of distributions supported at the origin of a Lie group. Using classical results of Poincaré and Schwartz, we identify it to the universal enveloping algebra of the Lie algebra of the group. Next, in the case of a compact Lie group, we introduce the algebra of so-called representative functions, and explain its structure as a corollary of the Peter-Weyl theorem. We then explain the relevance of these constructions for the theory of linear representations of a group. We also describe the duality between these two Hopf algebras.

The second hour of these lectures will be devoted to the abstract definitions connected with Hopf algebras, as well as to the three main structure theorems, namely :

a) The Tannaka-Krein reconstruction theorem, of a Hopf algebra from the "category" of its finite-dimensional representations.

b) The characterization of the enveloping algebras of Lie algebras.

c) The Hopf-Borel-Cartier-Milnor-Moore theorem asserting that certain algebras are free polynomial algebras, as well as a supersymmetric version.

We shall then proceed to some combinatorial applications , concerned with free Lie algebras, "shuffle products", Dyson chronological exponential. We shall show the relevance of these methods in various areas, from number theory to Quantum Field Theory.

- Y. Dokshitzer: I+II, 4hrs

Basics of perturbative QCD

The idea of compositeness of hadrons proved to be a major success of Quantum Field Theory (QFT) in application to the physics of "elementary" particles and their interactions. Three decades of Quantum Chromodynamics (QCD) were marked with spectacular achievements and, at the same time, plagued by deep conceptual problems. We know nowadays that the "coloured" quarks and gluons - the fundamental QCD fields - do exist, but are still incapable to explain how and why are they "confined" into colourless hadrons.

In the introductory lecture we will recall the general principles QTFs are built on, and discuss peculiarities of non-Abelian gauge fields - gluons: the origin of the "Asymptotic Freedom" being the key one.

Then, we will discuss in what extent, and in which sense, in the QCD framework one can control behaviour of quarks and gluons, measure their properties, and make verifiable predictions concerning hadrons.

In the third lecture a brief review will be given of the dynamics of "QCD partons" - i.e. gluons and quarks that multiply willingly in the so-called hard interactions involving large energy-momentum transfers.

Finally, we well touch upon recent theoretical developments that aimed at probing non-perturbative phenomena using perturbative concepts and tools, as well as at a better understanding of QCD with the help of super-symmetric QFTs.

- V. Rivasseau: I+II, 3hrs

Noncommutative quantum field theory

The first lecture will be devoted to a review of quantum field theory on noncommutative space of the Moyal type and its renormalization. In the second lecture I shall discuss some basic examples of constructive field theory techniques. Finally I'll review the relation between parametric representation of Feynman amplitudes in quantum field theory and multivariate Tutte polynomials.

- J. Zinn-Justin: I+II, 4hrs

Renormalization group: An introduction

Renormalization group has played a crucial role in 20th century physics in two apparently unrelated domains: the theory of fundamental interactions at the microscopic scale and the theory of continuous macroscopic phase transitions. In the former framework, it emerged as a consequence of the necessity of renormalization to cancel infinities that appear in a straightforward interpretation of quantum field theory and the possibility to define the parameters of the renormalized theory at different momentum scales.

In the statistical physics of phase transitions, a more general renormalization group was latter introduced to explain the universality properties of continuous phase transitions.

The field renormalization group now is understood as the asymptotic form of the general renormalization group in the neighbourhood of the Gaussian fixed point.

Correspondingly, we review here first the perturbative renormalization group, then a more general formulation called functional or exact renormalization group.

- J.-B. Zuber: I+II+III, 4hrs

  Combinatorial Applications of Matrix Integrals

  (1) Basics. Gaussian integrals and diagrammatic expansions

  (2) The one-matrix integral. Topological expansion. Computation techniques.

  (3) Connection with integrable systems (hints).

  (4) Combinatorics. Triangulations. Knots. Trees, etc.

- R. Flume: 1.5hr

  roundtabel session

Titles + Abstracts: Conference

Carlo ALBERT

Batalin-Vilkovisky Integrals via Homological Perturbation Theory

Abstract. The Batalin-Vilkovisky method (BV) is the most powerful method to analyze functional integrals with (infinite-dimensional) gauge symmetries presently known. Homological Perturbation Theory appears to be the adequate language to express a variety of concepts related to BV: BV-integrals and the BV-operator, open gauge symmetries, localization, anomalies and effective actions. In my talk, I will give an overview of these topics.

Marc BELLON

High order solution of a non-linear Schwinger-Dyson equation

Abstract. In the Hopf algebra approach to renormalization, the full propagator can be recovered from the renormalization group generator. This can be used to effectively compute the β-function from a non-linear Schwinger-Dyson equation. Indications are obtained for the singularity structure of the Borel transform of the obtained series. I shall also discuss the possible generalizations of these results.

Frédéric CHAPOTON

Operadic viewpoints on the Hopf algebra of rooted trees of Connes–Kreimer

Abstract. The Hopf algebra of rooted trees has been introduced by Connes and Kreimer some years ago, in their study of renormalization. It has also been useful in numerical analysis, in relation with the Butcher group. We will try to explain why this is also a natural structure in the context of operads, in two different ways.

Bertrand DELAMOTTE

An overview of the non perturbative renormalization group

Abstract. We show that the modern implementation of Wilson's renormalization group ideas is conceptually simpler than the usual perturbative formalism and leads to very powerful and non-perturbative methods in actual calculations. We illustrate this on several examples: calculations of universal and non-universal quantities in the O(N) model and in problems in out-of-equilibrium statistical mechanics such as the Kardar-Parisi-Zhang equation.

Bertrand DUPLANTIER

Liouville Quantum Gravity and KPZ

Abstract. We present a (mathematically rigorous) probabilistic and geometrical proof of the KPZ relation between scaling exponents in a Euclidean planar domain D and in Liouville quantum gravity. It uses a properly regularized quantum area measure in terms of the exponential of γ times the Gaussian free field on D. The proof extends to the boundary geometry. The singular case $\gamma > 2$ is shown to be related to the quantum measure for $\gamma' < 2$, by the fundamental duality $\gamma\gamma' = 4$.

Loïc FOISSY

Systems of combinatorial Dyson-Schwinger equations

Abstract. In the (commutative) Hopf algebra H of rooted trees decorated by the finite set $1, \ldots N$, with its 1-cocyles B_d^+ , we consider the following systems of Dyson-Schwinger equations:

$$X_d = B_d^+(f_d(X)),$$

where, for all d, $f_d(h)$ is a formal series in $h_1, \ldots h_N$, with coefficients in the base field. This system admits a unique solution $X = (X_1, \ldots, X_N)$ in a completion of H. The coefficients of X generate a subalgebra H_S of H. We here classifiy the formal series such that H_S is a Hopf subalgebra of H.

Alessandra FRABETTI

Combinatorial Hopf Algebras from renormalization

Abstract. I present three exemples of right-sided combinatorial Hopf algebras (in the sense of Loday and Ronco) which appeared in the Hopf algebraic approach to renormalization. It is a joint work with Christian Brouder.

José M. GRACIA-BONDÍA

Higgs-Mechanism-free Models

Abstract. In philosophical quarters, the Higgs(-Englert-Brout-Guralnik-Hagen-Kibble) mechanism is sometimes derided as a non-empirical device of doubtful explanatory value; it is often concluded that it possesses heuristic value in the "context of discovery", but less so, or none whatsoever, in the "context of justification". Since the unphysical fields involved in the Higgs mechanism are unobservable, this status question cannot be resolved by the likely sighting of Higgs particles in the LHC. Some current phenomenological puzzles make advisable a new look at the Higgs mechanism as well. We perform on it a "reality check" (historically the second) by studying renormalizability and gauge invariance in the framework of causal gauge theory (Epstein-Glaser renormalization). Which spontaneous symmetry breakdown-like models are allowed, and which disallowed, turns out to be an important internal question of the formalism. (Joint work with M. Duetsch, F. Scheck and J. C. Varilly.)

Razvan GURAU

Hunting bubbles in group field theory

Abstract. Group field theory is a generalization of matrix models. In 3 dimmensions its graphs are dual to 3D simplicial complexes. As such they are charcterized by several topological numbers: number of vertices N, lines L, faces F and bubbles B. In this talk we give the algorhitm to identify the bubbles of a graph and the associated genus g_B of a bubble. We furthermore give a direct proof ot the 3D Euler relation $N - L + F - B = -\sum_{bubbles} g_B$.

Stefan HOLLANDS

Perturbative quantum field theory in terms of vertex algebras

Abstract. Perturbative quantum field theory is the most important tool to obtain physical predictions about elementary particles, but it is also a remarkable mathematical structure. The main emphasis is mostly put on properties of scattering amplitudes and their relation to Feynman diagrams. In this talk, I want to emphasize the algebraic structure behind perturbative quantum field theory, which is ultimately tied to the operator nature of quantum fields. I explain how the framework of perturbative quantum field theory can be embedded into a higher dimensional version of so-called "vertex algebras", which in itself is a highly interesting algebraic structure. I will outline how this algebraic structure can be put into relation to the analytic aspect of perturbative calculations, i.e., to the kinds of special functions and series that come up there.

Kai Johannes KELLER

Euclidean Epstein-Glaser Renormalization

Abstract. In the framework of perturbative Algebraic QFT (pAQFT), recently developed by R. Brunetti, M. Duetsch and K. Fredenhagen, I will present a construction of "Euclidean time-ordered products", i.e. algebraic versions of the Schwinger functions, by adapting the recursive construction of Epstein and Glaser on Minkowski space to the Euclidean setting. In the introduced framework the Euclidean time-ordered product is the product of a partial algebra of functionals. Renormalization corresponds to extending the domain of definition of this product, wich is possible for a certain class of functionals, which are called local.

Thomas KRAJEWSKI

Some combinatorial aspects of Feynman diagrams evaluation

Abstract. After a brief review of renormalization, we introduce a general Hopf algebra of graphs and investigate the relationship between the universal topological polynomials for graphs in mathematics and the parametric representation of Feynman amplitudes in quantum field theory. We show how the Symanzik polynomials of quantum field theory are particular multivariate versions of the Tutte polynomial, and how the olynomials of noncommutative quantum field theory are particular versions of the Bollobas-Riordan polynomials. Furthermore, we sketch how the Mehler kernel propagators of non commutative field theories give rise to a new polynomial.

Govind KRISHNASWAMI

Large-N Matrix Models: some algebraic aspects

Abstract. Matrix models are quantum theories whose correlations are basis independent averages over the entries of several $N \times N$ matrices. They are toy-models for non-abelian gauge theories. In the 'classical' limit as N becomes large, various algebraic structures may be exploited to understand these models. For instance, the Schwinger-Dyson operators are invariant derivations of the shuffle-deconcatenation Hopf algebra. On the other hand, solving the Schwinger-Dyson equations involve computing the Legendre transform of a non-trivial one cocycle of the automorphism group of the free algebra.

Jean-Louis LODAY

Combinatorial Hopf algebras

Abstract. This is a joint work with Maria Ronco. We give a precise definition of "combinatorial Hopf algebras", and we unravel their structure in the four cases: associative or commutative, general or right-sided. For instance a cofree-cocommutative combinatorial Hopf algebra is completely determined by its primitive part which is a pre-Lie algebra. The key example is the Connes-Kreimer Hopf algebra, whose indecomposable algebra is the cofree pre-Lie coalgebra on one generator. Most examples of Hopf algebras arising in quantum field theory are combinatorial. The classification gives rise to several good triples of operads. It involves the operads: dendriform, pre-Lie, brace, and variations of them.

Dominique MANCHON

Composition and substitution: two Hopf algebras of trees interacting

Abstract. Motivated by recent work by Chartier, Hairer and Vilmart about composition and substitution of B-series in numerical analysis, we prove that the Connes-Kreimer Hopf algebra of rooted trees is a left comodule Hopf algebra over another Hopf algebra of rooted trees, graded by the number of edges.

Nikolay NIKOLOV

Anomalies in Quantum Field Theory and Cohomologies of Configuration Spaces

Abstract. In this work we study systematically the Euclidean renormalization in configuration spaces. We investigate also the deviation from commutativity of the renormalization and the action of all linear partial differential operators. This deviation is the source of the anomalies in quantum field theory, including the renormalization group action. It also determines a Hochschild 1-cocycle and the renormalization ambiguity corresponds to a nonlinear subset in the cohomology class of this renormalization cocycle. We show that the related cohomology spaces can be reduced to de Rham cohomologies of the so called "(ordered) configuration spaces". We find cohomological differential equations that determine the renormalization cocycles up to the renormalization freedom. This analysis is a first step towards a new approach for computing renormalization group actions. It can be also naturally extended to manifolds as well as to the case of causal perturbation theory. (preprint: arXiv:0903.0187)

Daniele ORITI

Group field theory: a microscopic description of quantum spacetime - some recent results

Abstract. We introduce the group field theory formalism for quantum gravity, a generalization of matrix models which incorporates ideas and structures of simplicial quantum gravity as well as loop quantum gravity. We then present some recent results concerning 1) the renormalization of a specific GFT model for 3d quantum gravity, 2) the emergence of non-commutative matter field theories from some simple GFT models in 3 and 4 dimensions.

Walter VAN SUIJLEKOM

Renormalization using Hopf algebras and the Batalin-Vilkovisky formalism

Abstract. I will report on the structure of renormalization Hopf algebras for gauge theories. Their coaction on the coupling constants is described in the context of Batalin-Vilkovisky (BV) algebras. The so-called master equation satisfied by the action in the BV-algebra implies the existence of certain Hopf ideals in the renormalization Hopf algebra.

Rainer VERCH

Dirac Field on Non-Commutative Spacetime

Abstract. We consider the quantized Dirac field on Moyal-Minkowski spacetime, serving as an example for field quantization on a general Lorentzian non-commutative spacetime. Observables are obtained from non-commutative potential scattering by way of Bogoliubov's formula. The talk is based on joint work with M. Borris, see arXiv:0812.0786[math-ph].

Fabien VIGNES-TOURNERET

Generalized Bollobas-Riordan polynomials and partial duality

Abstract. This talk is about ribbon graph invariants and their duality properties. After an introduction of a signed generalization of the Bollobas-Riordan polynomial and its relation with knot theory, I will present the new partial duality defined by S. Chmutov. It generalizes the usual duality which exchanges faces and vertices. I will finally define a multivariate version of the Bollobas-Riordan polynomial, invariant under the partial duality and give its main properties.