Shuffles, descents and representations.
A Conference in memory of Manfred Schocker
Nice, September 10–14.
Titles and abstracts.

- **Persi Diaconis'** Lecture Series : "Shufflings".
  - Lecture 1: "The mathematics of shuffling cards"

    Abstract: It takes seven ordinary riffle shuffles to mix up a deck of 52 cards. In addition to discuss this result obtained jointly with Dave Bayer, I will describe recent results (joint with Susan Holmes and Jason Fulman) on the analysis of casino shuffling machines (and the hyperoctaedral group) and results and open problems with less randomness required.
  - Lecture 2: "Descent algebras and quasi-symmetric functions"

    Abstract: This will be a gentle introduction to these topics, emphasizing their connection with riffle shuffling.
  - Lecture 3: "Mathematical appearences of riffle shuffling"

    Abstract: The basic shuffling combinatorics appears in many corners of mathematics. In this lecture, I describe Chen iterated integrals, work of Gerstenhaber-Schack et al. on Hochschild homology and work of Patras on geometric dissection problems. Even the basic work of carries in addition admits a shuffling interpretation.
  - Lecture 4: "Random walks on hyperplane arrangements"

    Abstract: A collection of hyperplanes in a Euclidean space breaks space into chambers. Bidigare-Hanlon-Rockmore have defined natural random walk on these chambers which includes a vast generalization of riffle shuffling. It also includes many natural real world processes (dynamic storage allocation and phylogenetic trees).

- **Marcelo Aguiar**: "Shuffles, the Coxeter complex of type A, and bilax monoidal functors"

- **Dieter Blessenohl** : "On the Canonical Lie Projector"

  Abstract: With respect to an appropriate metric the Malvenuto – Reutenauer algebra $\mathcal{P}$ possesses a completion $\widehat{\mathcal{P}}$, which is a convolution algebra. The canonical Lie projector R is a certain element of $\widehat{\mathcal{P}}$. Together with its convolution powers R induces via Polya action on free associative algebras the well known direct decomposition into symmetrized Lie powers. On the other hand, let $\mathfrak{A}$ denote the convolution subalgebra of $\mathcal{P}$, generated by the direct sum $\mathcal{O}$ of all right ideals $\omega_n K\mathcal{S}_n$, where ω_n denotes the Specht – Wever idempotent in the group algebra $K\mathcal{S}_n$ of the symmetric group $\mathcal{S}_n$. Then $\mathfrak{A}$ in an analogous way possesses a direct decomposition into symmetrized powers of $\mathcal{O}$. Further $\mathcal{O}$ is a Lie algebra under convolution, $\mathfrak{A}$ its universal envelope. A Poincaré – Birkhoff – Witt basis theorem holds for $\mathfrak{A}$. We conclude remarks on representations of the full linear group.

- **Roger Bryant** : "The module structure of free Lie algebras"

  Abstract: I shall describe joint work with Manfred Schocker that leads to a decomposition theorem for Lie powers in non-zero characteristic. Let V be a finite-dimensional module for a group G over a field K of prime characteristic p. The nth tensor power $T^n(V)$ is a module both for G and for the modular descent algebra of degree n. This leads to a decomposition of $T^n(V)$ as a direct sum of submodules indexed by the 'p-equivalence' classes of partitions of n. The summand indexed by the equivalence class J has a filtration with factors isomorphic to the 'higher Lie powers' corresponding to the partitions belonging to J.

- **Tomasz Brzezinski** : "Corings and comodules"

  Abstract: Corings are generalisations of coalgebras: they are bimodules (of not necessarily commutative ring) with comultiplication and counit. Motivated by their close connection to Hopf modules, a considerable progress in understanding the structure of comodules of corings has

been made recently. In this talk we will give an overview of this progress starting from basic definitions and examples. The talk is intended for a non-specialist audience.

- **Pierre Cartier** : "From multizeta values to free Lie algebras"

  Abstract: It is well-known that the multizeta values (MZV for short) satisfy two families of quadratic relations: the so-called SHUFFLES and STUFFLES relations. In the recent work of various authors (Ohno, Zagier, Ihara, Kaneko, Tanaka, Kawashima) a large number of linear relations among the MZV have been presented. We propose a synthesis of all these relations in the following form: it is shown that a certain generating series wuth the MZV as coefficients belong to a certain free Lie algebra already considered by Hofman. Our calculations rest on a multivariable generalization of the Hurwitz zeta function.

  Résumé : Les nombres multizetas (appelés aussi nombres d'Euler-Zagier , et désignés par le sigle MZV) satisfont à deux familles de relations quadratiques , dites de double mélange. Il est connu que les nombres multizetas satisfont à des relations linéaires qui ne sont pas conséquences des relations quadratiques précédentes. Un grand nombre de telles relations linéaires apparaissent dans les travaux récents de toute une série d'auteurs : Ohno, Zagier, Ihara, Kaneko, Tanaka, Kawashima. Nous proposons une synthèse de ces relations linéaires qui repose sur le résultat suivant : une certaine série génératrice non commutative, dont les coefficients sont les nombres muiltizetas, appartient en fait à une certaine algèbre de lie libre, déjà considérée dans l'étude du second produit de mélange. La démonstration utilise des généralisations à plusieurs variables de la fonction zeta d'Hurwitz.

- **Frédéric Chapoton** : "Shuffles and operads"

- **Mihai Ciucu** : "No-feedback card guessing for dovetail shuffles"

  Abstract: We consider the following problem. A deck of $2n$ cards labeled consecutively from 1 on top to $2n$ on bottom is face down on the table. The deck is given k dovetail shuffles and placed back on the table, face down. A guesser tries to guess at the cards one at a time, starting from top. The identity of the card guessed at is not revealed, nor is the guesser told whether a particular guess was correct or not. The goal is to maximize the number of correct guesses. We show that for $k \geq 2\log_2(2n)+1$ the best strategy is to guess card 1 for the first half of the deck and card $2n$ for the second half. This result can be interpreted as indicating that it suffices to perform the order of $\log_2(2n)$ shuffles to obtain a well mixed deck, fact proved by Bayer and Diaconis. We also show that if $k = c\log_2(2n)$ with $1 < c < 2$ then the above guessing strategy is not the best.

- **Kurusch Ebrahimi-Fard** : "Rota-Baxter type identities and shuffle products"

  Abstract: In this talk we review the construction of free Rota-Baxter type algebras using generalized shuffle products. Then we report on recent progress in the understanding of the role played by such algebras in several contexts, e.g. in differential systems and in the renormalization process of perturbative QFT (joint work with F. Patras, D. Manchon and J.M. Gracia-Bondia).

- **Karin Erdmann** : "The Lie module of the symmetric group "

  Abstract : (Joint work with K.M. Tan). We are interested in the projective part of the Lie module Lie(n) of the symmetric group $\mathcal{S}_n$ over fields of prime characteristic p. When $n = pk$ and p does not divide k, a joint result with Manfred Schocker parametrizes the non-projective part of Lie(n). We use this parametrization to find a bound for the dimension of the maximal projective part. This has a connection with algebraic topology.

- **Jason Fulman** : "Card shuffling and random walk on representations"

  Abstract: We give an introduction to random walk on the set of irreducible representations of a finite group. Applications to spectral theory and quantum computing are surveyed, and we describe how to obtain sharp convergence rate bounds for these walks. Connections with card shuffling are discussed.

- **Phil Hanlon** : "A Hodge decomposition of the coloring complex and coefficients of the chromatic polynomial"

  Abstract: Let G be a simple graph with n nodes. The coloring complex of G, as defined by Steingrimsson, has faces consisting of all ordered set partitions of n in which at least one block contains an edge of G. Jonsson proved that the homology of the coloring complex is concentrated in top degree and that the dimension of the homology of the coloring complex is one less than the number of acyclic orientations of G.

In this talk, we will show that the Eulerian idempotents give a Hodge decomposition of the top homology of the coloring complex of G into n-1 components. We go on to show that the dimensions of the Hodge pieces are equal to the absolute values of the coefficients of the chromatic polynomial of G. This gives a new algebraic interpretation for the coefficients of the chromatic polynomial.

- **Anthony Henderson** : "Orbit closures in the enhanced nilpotent cone"

  Abstract: It is well known that $GL_n(\mathbf{C})$-orbits in the nilpotent cone $\mathcal{N}_n$ (consisting of $n \times n$ nilpotent complex matrices) are parametrized by partitions of n. Some of the geometry of the orbit closures is reflected in their intersection cohomology polynomials, which were shown by Lusztig to equal the combinatorial Kostka–Foulkes polynomials. The 'enhanced nilpotent cone' of the title is nothing but the product $\mathbf{C}^n \times \mathcal{N}_n$ (consisting of pairs of a vector and a nilpotent matrix). The obvious action of $GL_n(\mathbf{C})$ still has finitely many orbits, now parametrized by bipartitions of n. I will discuss the closures of these orbits, and a conjecture on their intersection cohomology polynomials.

  This is joint work with Pramod Achar (Louisiana State University).

- **Christian Kassel** : "Some universal constructions on Hopf algebras"

  Abstract: (Joint work with Eli Aljadeff) This lecture is devoted to several universal constructions leading to interesting identities for each Hopf algebra. It includes a generalization of Dedekind's group determinant. We shall work out these constructions on the simple example of the Sweedler algebra, which is the smallest non-commutative non-cocommutative Hopf algebra.

- **Terry Lyons** : "The expected signature, shuffle products and Lévy area"

  Abstract: This talk will report on joint work of Daniel Levin and Mark Wildon. Consider the paths of finite length on a vector space V. The co-tensor algebra T(V*) is a natural core for the real-valued functions that are invariant under reparameterisation of paths. Pointwise multiplication of functions corresponds to shuffle products in the co-tensor algebra. This correspondence extends to the Brownian setting, and one can easily calculate the integral of any such function. In particular, one can compute the moments of Ly area quite explicitly. However, it seems to be a non-trivial combinatorial exercise to prove that these moments agree with the moments as described through the characteristic function of Ly area, described in terms of sec, as computed by Ly in his Berkeley symposium paper. This is a pity, because there are a number of very interesting situations where one would like to prove properties of measures on paths using these moments. At the moments, there seem to be significant combinatorial issues.

- **Jean-Yves Thibon** : "Commutative combinatorial Hopf algebras"

  Abstract: We describe some applications of a general method for constructing commutative (but in general non-cocommutative) Hopf algebras based on various sets of combinatorial objects. By duality, we recover all known enveloping algebra structures on such objects and several new ones.

- **Paul-Hermann Zieschang** : "On schemes which contain a constrained set of involutions, or: an algebraic approach to twin buildings"

  Abstract. Let X be a set, and let S be a partition of the cartesian product $X \times X$. Assume that $1_X \in S$ and that, for each element s in S, $s^* := \{(y, z) \mid (z, y) \in s\} \in S$. The set S is then called a scheme if, for any three elements p, q, and r in S, there exists a cardinal number a_{pqr} such that $|yp \cap zq^*| = a_{pqr}$ for any two elements y and z in X with $(y, z) \in r$.

  For any two elements p and q of a scheme S, we define pq to be the set of all elements s in S such that $1 \leq a_{pqs}$. A nonempty subset T of a scheme S is called closed if $p^*q \subseteq T$ for any two elements p and q in T.

  For each subset R of a scheme S, we define $\langle R\rangle$ to be the intersection of all closed subsets of S which contain R as a subset. An element s in a scheme S is called an involution if $|\langle\{s\}\rangle| = 2$.

  Let S be an association scheme, let L be a set of involutions of S, and assume that $\langle L\rangle = S$. The scheme S is called a Coxeter scheme with respect to L if it is constrained with respect to L and if L satisfies the exchange condition (a word by word translation of the group theoretic exchange condition to scheme theory).

It has been shown in [3; Theorem E] that Coxeter schemes can be identified with buildings in the sense of [1]. Moreover, from [4; Theorem 12.3.4] one knows that finite Coxeter schemes are quotients of thin schemes if they do not contain nontrivial thin elements and if the underlying set of involutions has at least three elements.

In my talk, I will discuss the question whether there exists a class of schemes which can be identified with twin buildings in the sense of [2] in a similar way as Coxeter scheme can be identified with buildings. If yes, would there be an analogue of the above-mentioned theorem on finite Coxeter schemes?

[1] Tits, J.: *Buildings of Spherical Type and Finite BN-Pairs*, Springer Lecture Notes in Math. **386**, Berlin Heidelberg New York (1974)

[2] Tits, J.: *Twin buildings and groups of Kac-Moody type*, London Math. Soc. Lecture Note Ser. **165**, Cambridge University Press (1992)

[3] Zieschang, P.-H.: *An Algebraic Approach to Association Schemes*, Springer Lecture Notes in Math. **1628**, Berlin Heidelberg New York (1996)

[4] Zieschang, P.-H.: *Theory of Association Schemes*. Springer Monographs in Mathematics, Berlin Heidelberg New York (2005)