On the validity of a nonlocal approach for MHD turbulence

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The phenomenology of magnetohydrodynamic (MHD) turbulence proposed by R. H. Kraichnan, and its consequences for energy spectra and energy decay laws in the high Reynolds number self-similar regime, are first reviewed. After recalling the exact relationships for MHD derived in the spirit of the so-called ''4/5'' law of Kolmogorov, such laws are used to compute intermittency exponents for statistically steady MHD flows in two space dimensions. These exponents are evaluated both for the structure functions of order p built on the physical variables—velocity $\mathbf{v}$ and magnetic field $\mathbf{b}$, or their combination, through the Elsässer variables $\mathbf{z}^{\pm}=\mathbf{v}\pm\mathbf{b}$—and for the flux variables $Y_p^{\pm}(r)\equiv\langle[\delta z_L^{\mp}(\mathbf{r})(\delta z_i^{\pm}(\mathbf{r}))^2]^{p/3}\rangle$, where L denotes the longitudinal components of the fields. There are indications that these nonlinear fluxes scale in a manner similar to that of nonconducting fluids, whereas the physical $\mathbf{v}$, $\mathbf{b}$, $\mathbf{z}^{\pm}$ fields are more intermittent than in the neutral fluid case. © *1999 American Institute of Physics.* [S1070-6631(99)01608-6]

I. INTRODUCTION

Many astrophysical plasmas, e.g., in the solar wind or the interstellar medium, are observed in a turbulent state with velocities substantially smaller than the speed of light. In the magnetohydrodynamic (MHD) approximation thus relevant for describing the dynamics of large scales, will the Kolmogorov approach prevail, or are the classical scaling laws of turbulent fluids modified because one does have to take into account the specificity of MHD interactions that appears for example through the propagation of Alfvén waves? In that sense, MHD can be viewed as a testing ground as to the universality of turbulent scaling. In particular, when restricting the analysis to incompressible flows, we first consider in this paper what corresponds for conducting fluids to the three relations derived by Kolmogorov in 1941[1–3] (henceforth K41) and deal respectively with the self-similar scaling of the energy spectrum in Fourier space $E(k)$, the temporal energy decay $E(t)$ and the exact scaling laws followed by third-order structure functions.

The first scaling law of Kolmogorov concerning the energy spectrum can be viewed simply as a dimensional statement, once the premise—that of the existence of an inertial range within which the transfer of energy to small scales occurs with a constant energy flux ϵ—is given; it leads to the isotropic kinetic energy spectrum $E(k)=C_K\epsilon^{2/3}k^{-5/3}$ with C_K the Kolmogorov constant. The second scaling law deals with the temporal energy decay in a temporal self-similar regime. It relies on the phenomenological assumption of the persistence of large-scale eddies together with assuming (i) a formulation of the energy transfer consistent with the energy spectrum quoted above, namely $\epsilon\sim v^3/\ell_0$ [with ℓ_0 the integral scale and v the root-mean-square (r.m.s.) velocity fluctuation], and (ii) a power-law dependence of the energy spectrum in the large scales $E(k)\sim k^4$. The decay law, once small scales are formed at t_*, reads $E(t)\sim(t-t_*)^{-10/7}$. Finally, the third Kolmogorov scaling law is of a different nature since it deals with an exact relationship for the third-order longitudinal structure function of the velocity field in the inertial range, assuming isotropy, homogeneity, incompressibility and stationarity; it reads $\langle\delta v_L^3(\mathbf{r})\rangle=-(4/5)\,\epsilon\,r$, where brackets indicate space-averaging and $\delta v_L(\mathbf{r})=[\mathbf{v}(\mathbf{x}+\mathbf{r})-\mathbf{v}(\mathbf{x})]\cdot\hat{\mathbf{r}}$ is the longitudinal velocity increment for the scale separation $r=|\mathbf{r}|$, with $\hat{\mathbf{r}}=\mathbf{r}/r$ the unit vector along $\mathbf{r}$. An extension that includes all components of the velocity field is performed by Fulachier and Dumas.[4]

How general are such laws? Except for the third one, the equations of motions are not used but for an evaluation of characteristic times and a realization that the nonlinear terms induce mode-coupling and transfer to small scales. The self-similarity assumption (of the first kind, see, e.g., the monograph of Barenblatt[5]) then allows us to deduce simple scaling laws. This paper addresses the question of the universality of the Kolmogorov laws in the more complex framework of incompressible MHD. Now, there are waves propagating in this flow inducing a degree of anisotropy, as well as nonlocality of the nonlinear interactions, because such waves propagate along the lines of a strong uniform (or quasi-uniform) magnetic field.

The next section reviews the phenomenology developed in this context for the energy spectrum of MHD turbulence,[6,7] the ensuing temporal decay law of energy that can be deduced using the same phenomenology, and finally the exact laws stemming from the conservation properties of the MHD equations in the absence of dissipation. Section III gives the main characteristics of the two-dimensional driven flows dealt with in this paper, and Sec. IV is devoted to the

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intermittent properties of such flows. Finally, Sec. V provides a discussion and a conclusion.

II. A NONLOCAL PHENOMENOLOGY FOR MHD IN THE PRESENCE OF ALFVÉN WAVES

In this section, we review the phenomenology developed by Kraichnan[6] (see also Iroshnikov[7]) for incompressible MHD flows. In the incompressible case, the equations read

$$(\partial_t+\mathbf{v}\cdot\nabla)\mathbf{v}=-\nabla P+\mathbf{j}\times\mathbf{b}+\nu\nabla^2\mathbf{v}+\mathbf{F^V}, \tag{1}$$

$$(\partial_t+\mathbf{v}\cdot\nabla)\mathbf{b}=\mathbf{b}\cdot\nabla\mathbf{v}+\eta\nabla^2\mathbf{b}+\mathbf{F^M}, \tag{2}$$

$$\nabla\cdot\mathbf{v}=0, \quad \nabla\cdot\mathbf{b}=0, \tag{3}$$

where $\mathbf{v}$ and $\mathbf{b}$ are the velocity and the Alfvén velocity with $\mathbf{b}=\mathbf{B}/\sqrt{\rho_0\mu_0}$, where $\mathbf{B}$ is the magnetic induction, ρ_0 the mean uniform density and μ_0 the permeability; P is the pressure, $\mathbf{j}=\nabla\times\mathbf{b}$ the current density associated with the Alfvén velocity; ν and η are, respectively, the viscosity and magnetic diffusivity, and $\mathbf{F^{V,M}}$ are driving forces to be specified later (see Sec. III A).

A. The energy spectrum of MHD turbulence

Concerning the energy spectrum of MHD turbulence, Kraichnan[6] (see also Iroshnikov[7] for an independent statement) proposed a model (henceforth called IK) for which the inertial index of the energy spectrum is modified from its Kolmogorov value because energy transfer to small scales is less efficient in the presence of a large-scale magnetic field $\mathbf{B_0}$ (more precisely, the magnetic induction). This model was further studied in the context of two-point closures of turbulence by Kraichnan and Nagarajan[8] in the full isotropic case. Indeed $\mathbf{z}^{\pm}=\mathbf{v}\pm\mathbf{b}$ Elsässer eddies travel in opposite directions along the lines of $\mathbf{B_0}$ and interact only when they meet. In the case of negligible mean correlation between the velocity and the magnetic field, this results in an energy spectrum $E_{\mathrm{MHD}}(k)=C_{\mathrm{IK}}(\epsilon_B B_0)^{1/2}k^{-3/2}$ (with B_0 the amplitude of $\mathbf{B_0}$) as opposed to $E_{NS}(k)\sim k^{-5/3}$ for K41. This spectral law can be obtained from dimensional analysis, once the slowing down of the energy transfer, due to the presence of Alfvén waves, is incorporated in a model for the characteristic transfer time τ_{tr}, namely

$$\tau_{\mathrm{tr}}=\tau_{NL}[\tau_{NL}/\tau_A], \tag{4}$$

where $\tau_{NL}=1/\sqrt{k^3E_{\mathrm{MHD}}(k)}$ and $\tau_A=(kB_0)^{-1}$ are respectively the eddy turnover time and the Alfvén time. This different spectral law stems from the fact that interactions are nonlocal, a large-scale magnetic field having a nontrivial effect on the dynamics, whereas a large-scale velocity field is eliminated through Galilean invariance.

Note that the anisotropy that results from the presence of a strong uniform field $\mathbf{B_0}$, with the turbulence mostly confined to the plane perpendicular to $\mathbf{B_0}$ as shown in several numerical simulations,[9–11] can be studied as well in the framework of the weak turbulence formalism[12] that leads to a set of coupled integrodifferential ''kinetic'' equations[13] at the level of three-wave interactions. In that approximation—and in a wave-packet formalism developed in Refs. 14 and 15 as well—there is no energy transfer along the uniform magnetic field. Furthermore, the resulting turbulence in the orthogonal plane is not of the Kolmogorov type,[13,15] contrary to what is stipulated by Goldreich and Sridhar.[16] This spectral dependence can be obtained[13] both analytically by a local analysis, and numerically using exponential discretization of the kinetic equations in Fourier space as is customary for closure equations.

The difference between the numerical values of the inertial range indices of the IK and K41 spectra is small. Moreover, one should take into account for both that there likely will be intermittency corrections; this point is further discussed in Secs. III C and IV A. Finally, the presence of strong correlations between the velocity and the magnetic field may alter as well the spectral laws just discussed; this was shown phenomenologically with two-point closures.[17] Thus, a direct numerical confirmation of the IK phenomenology for energy spectra in MHD is somewhat difficult, although numerical results for two-dimensional flows[18] indicate a variation of the spectral indices with the amount of total velocity–magnetic field correlation present in the flow, in a way that is consistent with the IK model modified to take into account the presence of such a correlation.

B. The temporal decay of MHD turbulence

The self-similar temporal energy decay in MHD is obtained in a fashion similar to the K41 case, using the persistence of large-scale eddies together with the IK-compatible phenomenological relationship between the rate of energy transfer ϵ_B evaluated as $dE_{\mathrm{MHD}}/dt\sim E_{\mathrm{MHD}}/\tau_{\mathrm{tr}}$, the integral scale ℓ_0 and the amplitude z of the $\mathbf{z}^{\pm}$ fields, viz. $\epsilon_B\sim z^4/(\ell_0 B_0)$, assuming negligible velocity–magnetic field correlation, an assumption leading to $z^+\sim z^-\sim z$. In the standard case of a large-scale spectrum following a k^{D+1} law in dimension D, the energy decay law is $\sim(t-t_*)^{-5/6}$ for $D=3$ and $\sim(t-t_*)^{-4/5}$ for $D=2$.[19] In the three-dimensional (3D) case, this decay is very close to a t^{-1} law predicted on different grounds;[20] thus, numerical simulations in 3D will hardly be able to differentiate between models. On the other hand, the two-dimensional geometry allows for more numerical resolution, and the ensuing results seem clear. Indeed, the proposed model[19]—and its generalization[21] by the use of a parameter governing the wave number dependence of the energy spectrum at scales of the order of, or larger than, the integral scale— lead to predictions that are backed up by a series of two-dimensional (2D) numerical simulations. However, in the case of strong velocity–magnetic field correlations, the results are more ambiguous, but in all cases an overall transfer occurs that is slower than for neutral fluids: the observed decay of energy behaves in a way that favors for MHD flows the IK phenomenology as opposed to the Kolmogorov one.

C. Exact laws for MHD turbulence

The third Kolmogorov law, also called the ''4/5'' law, and its generalization,[4] are exact; they stem from the conservation properties of the fundamental equations by the nonlinear terms, assuming an inertial range together with incompressibility, isotropy, homogeneity and stationarity. The two equivalent MHD laws have been derived recently[22,23]: linear scaling with distance obtains as well in MHD for third-order correlators involving the Elsässer variables $\mathbf{z}^{\pm}$. For example, one can use a formulation first introduced by Yaglom[24] in the case of the passive scalar (see also Fulachier and Dumas[4] for the neutral fluid case). This relation in MHD involves the total energies $\langle z_i^{\pm 2} \rangle$. Defining the ''Yaglom'' fields

$$Y_p^{\pm}(r) = \langle [\delta z_L^{\mp}(\mathbf{r})(\delta z_i^{\pm}(\mathbf{r}))^2]^{p/3} \rangle, \tag{5}$$

one obtains in dimension d,[23]

$$Y_3^{\pm}(r) = \langle \delta z_L^{\mp}(\mathbf{r})(\delta z_i^{\pm}(\mathbf{r}))^2 \rangle = -\frac{4}{d}\epsilon^{\pm} r, \tag{6}$$

where $\delta z_i^{\pm}(\mathbf{r}) = z_i^{\pm}(\mathbf{x}+\mathbf{r}) - z_i^{\pm}(\mathbf{x})$, the subscript L denotes as before the longitudinal component of the fields, summation over the repeated index i is understood, and $\epsilon^{\pm}$ are the mean energy transfer and dissipation rates of the $\mathbf{z}^{\pm}$ fields. Both sets of relationships can be used to obtain a better evaluation of anomalous exponents of high-order structure functions for MHD flows,[25,26] following the Extended Self-Similarity procedure[27] (henceforth called ESS), i.e., replacing the independent variable r by $Y_3^{\pm}(r)$.

Finally, note that the extension of the K41 phenomenology to the scaling of higher-order structure functions, namely $\langle \delta v_L(\mathbf{r})^p \rangle \sim r^{p/3}$, agrees at third order with the ''4/5'' law. However, the MHD case is more complex because it involves the correlation between the $\mathbf{z}^{\pm}$ variables, and similarly between the velocity and the magnetic field.

On the one hand, the equivalent generalization of the IK phenomenology leads to a linear behavior of exponents for the structure functions of the Elsässer variables. Defining as usual longitudinal structure functions of order p as

$$S_p^{\pm}(r) \equiv \langle (\delta z_L^{\pm}(\mathbf{r}))^p \rangle \sim r^{\zeta_p^{B\pm}}, \tag{7}$$

and writing in the inertial range that such functions possess power-law scaling with distance, one arrives at $\zeta_p^{B\pm} = p/4$ (see, e.g., Biskamp[28]), assuming that there is little correlation between the $\mathbf{z}^{\pm}$ variables and hence that the $\mathbf{z}^{\pm}$ fields may be scaling in identical ways. Hence the relationships (6) would lead to $\zeta_3^{B\pm} = 1$, in contradiction with the IK phenomenology (see also Politano *et al.*[25]). A resort to numerical simulations is thus needed in order to investigate the behavior of such structure functions with distance in MHD.

III. NUMERICAL EXPERIMENTS IN TWO DIMENSIONS

We now report on a series of numerical simulations using a pseudospectral method for the incompressible MHD equations in two dimensions, with periodic boundary conditions.

A. Large-scale spatio-temporal behavior

The time-dependent forcing in Eqs. (1) and (2) consists of maintaining at a constant level the amplitudes of a number of Fourier modes (see below) for both the velocity and the magnetic field. The total energy input per unit time is equal to 2.0 and the normalized correlation of the driving is typically between 1% and 4%.

TABLE I. List of the runs described in this paper, where N is the number of grid points per linear dimension, R is the integral Reynolds number based on the r.m.s. velocity field and $T_{\max}$ is the maximum time reached in the computation in units of the eddy turnover time τ_{NL} for that computation, once a statistically steady state has been reached. The results concerning anomalous exponents for Run A (Ref. 26) and Run B (Ref. 25) have already been reported in part.

Run	N	R	$T_{\max}$
A	512	170	800
B	1024	1600	160
C	512	780	280
D	1024	950	230

Two series of runs have been performed (see Table I) which differ in the range of scales that are driven. In the first case, the computations are performed on a grid of either 512^2 points (Run A), maintaining constant the amplitudes of all Fourier modes with $|\mathbf{k}|=2$ for about 800 eddy turnover times τ_{NL}, or on a grid of 1024^2 points (Run B), keeping constant all the modes with $|\mathbf{k}|=1$ for approximately 160 τ_{NL}.

These two runs stem from different compromises, between temporal and spatial resolution. At the lower spatial resolution, the flow is computed for long times and is statistically steady with strong energetic bursts, each spanning of the order of $300\tau_{NL}$; the inertial range is confined to at best a factor two in scale, insufficient for a precise determination of scaling exponents. This is due in particular to the fact that the lowest Fourier shell contains a negligible amount of energy because of the too high degree of symmetry of the forcing, making the integral Reynolds number effectively achieved in the statistically steady state too low (fluctuating around $R \sim 170$). On the other hand, at the higher spatial resolution, with a resulting Reynolds number of the order of 1600, the flow can only be computed for one such large energetic burst, i.e., it cannot be considered as statistically steady, although the data are computed for a time long enough in order to allow for a steady state to establish at small scales as diagnosed on the temporal evolution of the fluctuations in the total enstrophy. These long-time large fluctuations in the energy are due in part to the small number of modes that are excited.

In order to explore the genericity of the results obtained with the runs just described,[25,26] we have performed new computations with a slightly different type of driving, namely that all the Fourier modes in the first two shells are kept constant. Spatial resolutions are identical as in the previous cases, but with now a non-negligible amount of energy in the first Fourier shell, ensuring a more energetic stirring with a shorter span between large-scale energetic bursts.

B. Dynamics of the flows

Extensive tests indicate that the resulting flows in these new runs are now stationary on average for both the run on a grid of 512^2 points (Run C) and for the high resolution run

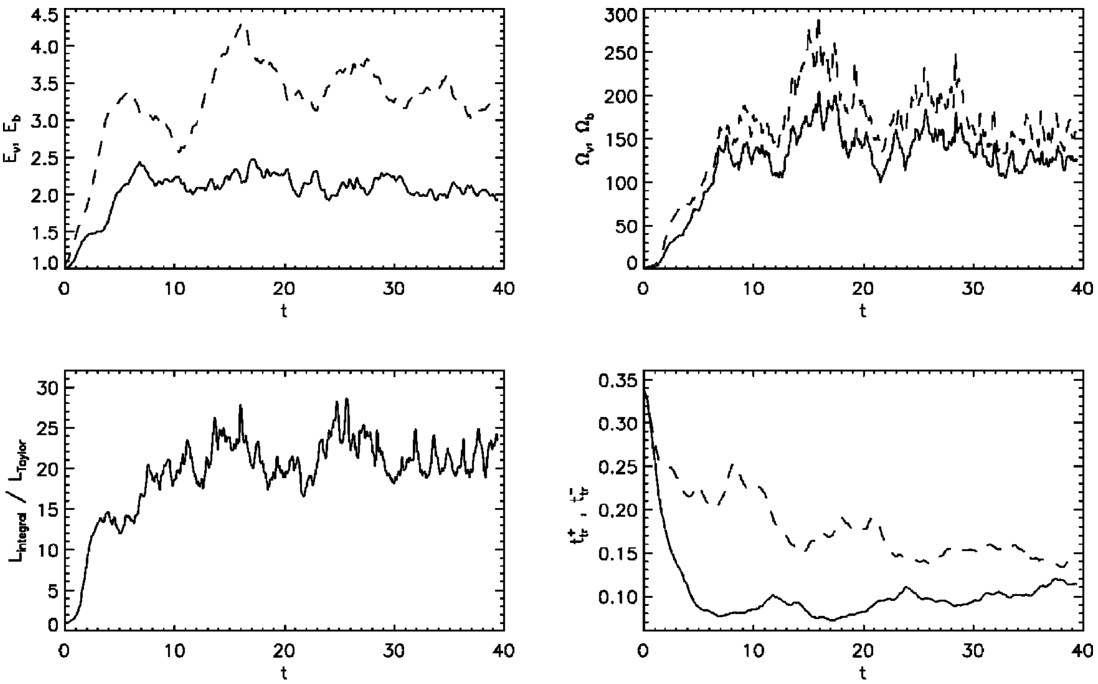

FIG. 1. Temporal evolution of the kinetic and magnetic energies (upper left), enstrophies (upper right), the ratio of the integral to the Taylor scale computed on the r.m.s. velocity field (lower left) and finally the transfer time of the ± variables (lower right). In the first two cases, a solid line represents the kinetic quantities, and the dashed line its magnetic counterpart, whereas in the last case these lines correspond, respectively, to the ± fields.

on a grid of 1024^2 points (Run D). The steady flow computed on a grid of 512^2 points is followed for 280 eddy turnover times, and that on a grid of 1024^2 points for roughly the same time. During that interval, the integral Reynolds number based on the r.m.s. velocity fluctuates around 780 for Run C and 950 for Run D. The correlation coefficient is of the order of 16% and its unsigned counterpart computed with the absolute value of the correlation coefficient at each point $2\langle|\mathbf{v}\cdot\mathbf{b}|\rangle/\langle|\mathbf{v}|^2+|\mathbf{b}|^2\rangle$ is $\sim 48\%$ for both flows. The ratio of the magnetic to kinetic energy remains of the order of 1.6 throughout the runs, i.e., with a slight excess of magnetic energy, as observed in many instances, e.g., in the solar wind and in the interstellar medium—a fact probably related to the local structure of magnetic X-points.[29] This discrepancy is less marked in the small scales, where the ratio of magnetic to kinetic enstrophy is equal on average to 1.25.

We show in Fig. 1 for Run D the temporal evolution of the kinetic and magnetic energies E_v and E_b (upper left), the kinetic and magnetic enstrophies $\Omega_v = \langle \boldsymbol{\omega}^2 \rangle$ and $\Omega_b = \langle \mathbf{j}^2 \rangle$ (upper right), the ratio of the integral to the Taylor scale ℓ_I/ℓ_T computed on the r.m.s. velocity field (lower left) with, respectively, $\ell_I = E_v^{-1}\int[E_v(k)/k]dk$ and $1/\ell_T = \sqrt{E_v^{-1}\int k^2 E_v(k) dk}$, and finally the transfer time of the ± variables (lower right) defined as in (4). In the first two cases, a solid line represents the kinetic quantities, and the dashed line its magnetic counterpart, whereas in the last case these lines correspond, respectively, to the ± Elsässer fields. Note the more bursty behavior of the magnetic energy, with a total of four such events each lasting of the order of 80 τ_{NL}.

Figure 2 displays, for the same run at $t=39.43$, the energy spectra of the Elsässer variables (top), and (bottom) the flux of total energy $\langle|\mathbf{z}^+|^2+|\mathbf{z}^-|^2\rangle$; both are in log–log coordinates. The fact that the $E^{\pm}$ energies are everywhere comparable throughout the range of wave numbers indicates that the velocity–magnetic field correlation, proportional to E^+

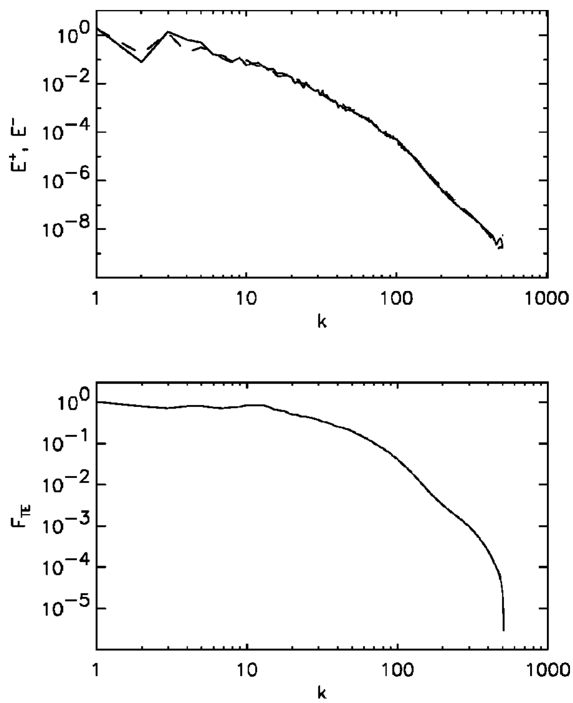

FIG. 2. Spectra of the ± energies (top) and of the flux of total energy (bottom) at $t=39.43$, in log–log coordinates for Run D with, on the top, $E^+(k)$ (solid line) and $E^-(k)$ (dash). Note that the flux is constant up to $k \sim 20$.

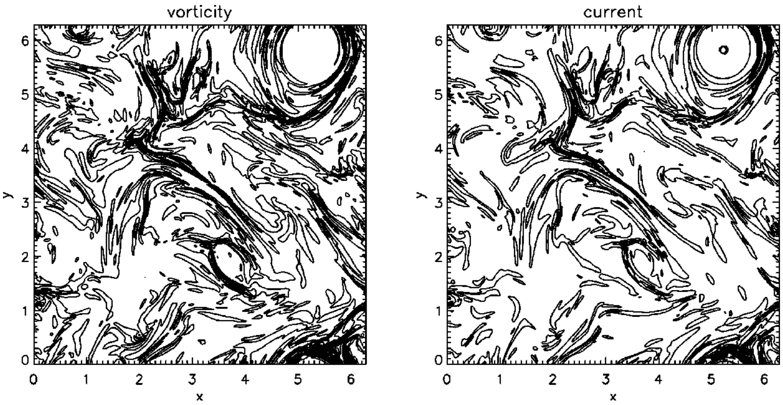

FIG. 3. Contours of vorticity (left) and current (right) at $t=39.43$ for Run D. Note the strong coherent structure in both fields in the upper right corner.

$-E^{-}$, is negligible at all scales in this flow. From the spectra and the domain where the flux is almost constant outside the driving range, we deduce that the inertial range extends approximately from $k=5$ to $k=20$. Also note the dynamics of the computation, with the spectra down to 10^{-8} in the smallest scales. Finally, Fig. 3 gives the contours of the vorticity (left) and current (right) at the same time as for Fig. 2. Note in both fields the presence of sheets as well as a large coherent structure on the upper east side corner.

IV. INTERMITTENCY IN MHD FLOWS

Both temporal and spatial intermittency is known to occur in MHD flows. For example, the heating of the solar corona is strongly variable, and the resulting histograms of the energy output of the flares (see, e.g., Galtier and Pouquet[30] and references therein) follow a power law on several decades. Such histograms can be recovered with one-dimensional MHD in which the footpoint motion of magnetic arches—because of photospheric convection—is modeled by a simple time-varying driving.[26]

Spatial intermittency obtains as well in turbulent MHD flows, with strongly concentrated current and vorticity sheets. However, there are up to now few computations of anomalous exponents of structure functions for MHD. In Grauer and Marliani,[31] a K41 scaling is assumed (namely, $\zeta_3^{B\pm}\equiv 1$), and furthermore the computations are done for a decaying flow and are thus restricted to a small statistical set. In the solar wind,[32,33] anomalous exponents of structure functions have been measured as well, but with too few data points (of the order of a few thousand) to be reliable much beyond the third order (see also Ref. 34).

Here, we tackle the problem of characterizing quantitatively the intermittent structures in MHD for the driven flows described in the preceding section. Taking large samples using averaging over a total time spanning more than 210 τ_{NL}, and furthermore using the exact relationship of Eq. (6) in order to obtain scaling laws on the largest possible range of scales, we compute structure function exponents up to $p=8$. Computing displacements in both the x and y directions leads finally for Run D to a total of $\sim 1.2\ 10^8$ data points.

Figure 4 shows in log–log coordinates the $Y^{\pm}$ correlators (on the top), with $Y^{\pm}(r)=\langle|\delta z_L^{\mp}(\mathbf{r})|(\delta z_i^{\pm}(\mathbf{r}))^2\rangle$ (differing from $Y_3^{\pm}$ by the use of absolute values) as a function of r/L_0, where $L_0=2\pi$ is the length of the computational box. A linear behavior is visible extending approximatively from $r/L_0=0.1$ to 0.17 for Y^{+}, and from $r/L_0=0.08$ to 0.2 for Y^{-}. This gives a measure of the extent of the inertial range where such a behavior is expected according to the rigorous result (6); this estimate of the domain of the inertial range is compatible with the examination of the spectra and the zone of constant flux (see Fig. 2). On the bottom, Fig. 4 displays the same correlators $Y^{\pm}$ now computed on a reduced sample of size $\sim 7.2\ 10^7$ data points obtained by avoiding the strong coherent structure of the vorticity and the current, visible on Fig. 3 (upper right corner). This results in limiting the influence of the large scales on the structure functions $Y_p^{\pm}(r)$, in particular on the $Y^{\pm}$ correlators leading to an enlarged linear scaling range for these variables. This linear behavior is now

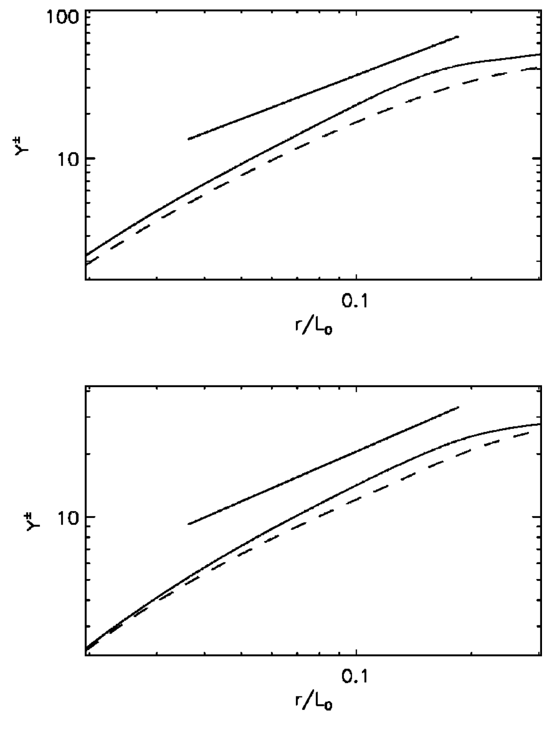

FIG. 4. Log–log plot of the correlators $Y^{+}(r)$ (solid line) and $Y^{-}(r)$ (dash) versus r/L_0 for Run D; a line with a slope of one is also shown for comparison. The correlators are computed with the full sample on the top, and with the reduced sample avoiding the large coherent structure on the bottom.

extending in the range $0.08 \leqslant r/L_0 \leqslant 0.2$ for Y^+ and in the range $0.06 \leqslant r/L_0 \leqslant 0.22$ for Y^-; the increased scaling range occurs as well for high orders, as checked up to $p=8$. The study of the influence of large-scale coherent structures on the intermittency properties of the flow is left for future work.

A. Intermittency for the Elsässer variables

It is known that solar wind data stemming from the Voyager spacecraft give for the observed magnetic energy spectrum[35] an inertial index very close to the Kolmogorov one, viz. $\zeta_2^{B\pm} \sim 0.67$. But the interpretation may not be as straightforward as it seems: intermittency for neutral fluids leads to a spectrum somewhat steeper than the Kolmogorov law (namely, ~ 0.70). Using Run A, it was shown[25] that the second-order structure functions of the Elsässer fields have a scaling exponent close to 0.70; but this fortuitous coincidence at second order does not arise for higher orders. Hence, it is essential that one be analyzing high-order structure functions of conducting flows in order to be able to clearly discern fluidlike from wavelike effects in MHD turbulence. The results[25] indicate that the measured scaling exponents for the structure functions of the Elsässer variables display a stronger departure from a linear scaling law than in the fluid case, thus evidencing a stronger intermittency in MHD. Furthermore, neither the She–Leveque model (henceforth SL)[36] nor its extension to MHD[31,37] work for the data analyzed in Ref. 25. However, these results are obtained from Run B, with temporal averaging over only one energetic burst, i.e., with insufficient large-scale temporal sampling.

We thus now analyze Run D for which, as stated before and as displayed in Fig. 1, several energetic bursts are covered by the simulation. The results are given in Fig. 5, which gives $\xi_p^{B\pm}$ as a function of the order p for the Elsässer fields as defined in (7), using in the ESS procedure the flux relations (6), viz.,

$$\langle | \delta z_L^{\pm}(\mathbf{r}) |^p \rangle \sim [Y^{\pm}(r)]^{\xi_p^{B\pm}}. \tag{8}$$

The broken line represents the She–Leveque model, and the dotted line is the $p/3$ Kolmogorov linear scaling law. The variations on the computed exponents for the + variables range from ± 0.005 to ± 0.4, the larger for larger p, and from ± 0.005 to ± 0.4 for the − variables. Thus, the computations performed on Run D confirm the strong intermittency of the Elsässer fields found with Run B.

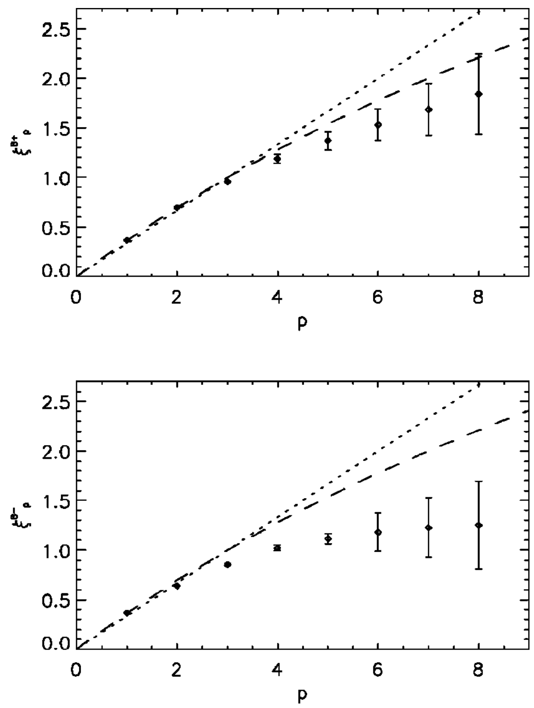

FIG. 5. Anomalous exponents $\xi_p^{B\pm}$ [see relation (8)] with error bars, as a function of order p for Run D; also shown are the K41 law (dotted line) and the SL model for fluid flows (dashed line). On the top (resp. bottom), is given ξ_p^{B+} (resp. ξ_p^{B-}) with error bars ranging from ± 0.005 to ± 0.4 (resp. from ± 0.001 to ± 0.4).

B. Intermittency for the flux variables

1. Analogy with the passive scalar

A passive scalar such as the temperature field θ is known to be more intermittent than the velocity that is advecting it, in the sense that its structure functions $S_p^{\theta}(r) = \langle | \delta\theta(\mathbf{r}) |^p \rangle \sim r^{\zeta_p^{\theta}}$ have scaling exponents that depart more strongly from a linear law than the fluid exponents for the velocity structure functions. However, there are several indications—both numerical and experimental[38–40]—that the flux variables $Y_p^{\theta}(r) = \langle [| \delta v_L(\mathbf{r}) | \delta\theta(\mathbf{r})^2]^{p/3} \rangle$ scale differently: it is found[38,39] that the structure functions of the flux scale quite similarly to the structure functions of the velocity that carries along the scalar.

We now address the similar question for MHD flows, namely what is the scaling of the flux functions defined in (5)? Will the propagation of Alfvén waves—which is known to modify the transfer of energy to small scales by slowing it down, as encompassed in the IK model—induce yet a different scaling behavior for the Yaglom fields $Y_p^{\pm}$? In other words, does the nonlocal character of MHD flows induce a modification of the scaling of the flux variables introduced in Eqs. (5), when compared to either the scaling of the structure functions of the basic physical ($\mathbf{v}$, $\mathbf{b}$, $\mathbf{z}^{\pm}$) fields, or when contrasted to the fluid case?

If it does alter it, it means that MHD effects imply a complete loss of universality as far as scaling is concerned. If, on the other hand, the scaling of flux variables is identical for fluid and for MHD flows—when the scaling of the basic fields differ, as shown in Politano *et al.*[25] and in the preceding Section—it might mean that this slowing down of transfer to small scales only modifies time scales and the amount of dissipation of the energy (the transfer being less efficient), but that the anomalous scaling observed for fluids—and modeled, e.g., by She and Leveque[36]—and for MHD (at least in two dimensions) is linked with the advection term common to all these equations. In which case, the fluid scaling of velocity structure functions should have a large degree of universality, and be observed as well in other flows, as a signature of advection by the flow. It is already known that supersonic flows maintained by a combination of three orthogonal shear waves with a resulting r.m.s. Mach number of unity have anomalous scaling exponents close to those of incompressible fluids,[41,42] with numerous strong shocks present in the flow.

2. Numerical results

The scaling laws for the flux variables in MHD have already been computed for Run A,[26] a run which is at a lower Reynolds number than Run B but which is statistically steady. These results indicate that the Yaglom fields scale as in the fluid case, not like the MHD variables $\mathbf{z}^{\pm}$. It is the main purpose of this paper to progress in this investigation by analyzing a flow which is both at a reasonably high Reynolds number (higher by more than a factor five than for Run A) and which is also statistically steady. We thus now examine the intermittency properties of Run D on the flux variables.

As stated before, a total data set of $\sim 1.2\ 10^8$ points is analyzed; it corresponds to 56 temporal snapshots for Run D, each snapshot being separated by three or four turnover times. We checked that a temporal separation ten times larger between snapshots did not change the results reported here, when taking a subsample. Displacements are taken both in the x- and y-directions, leading to a more isotropic data set than in the previous study of Run A.[26]

The isotropy of the resulting flow is measured through the following coefficients: $\sqrt{z_x^{\pm 2}/z_y^{\pm 2}}$ measuring isotropy at large scale, and $\sqrt{(\nabla\times\boldsymbol{\omega}^{\pm})_x^2/(\nabla\times\boldsymbol{\omega}^{\pm})_y^2}$ (with $\boldsymbol{\omega}^{\pm}=\boldsymbol{\omega}\pm\mathbf{j}$, where $\boldsymbol{\omega}$ is the vorticity and $\mathbf{j}$ the current) measuring isotropy at small scale. When averaged over the 56 snapshots, both are found to vary around 1.10 for the $\pm$ fields; note that for a data set half the size with 28 snapshots in the statistically steady regime, these coefficients are approximately equal to 1.20.

The anomalous exponents of higher-order structure functions are computed, using the data plotted in Fig. 4 (top) to calibrate the inertial range. Figure 6 displays, on the top, the exponent ξ_p^+ as a function of the order p, where

$$Y_p^+(r)\sim[Y^+(r)]^{\xi_p^+}, \tag{9}$$

using absolute values in the evaluation of the $Y_p^{\pm}$ fields, and taking advantage of the so-called ESS procedure[27] adapted to the MHD problem, i.e., replacing the independent variable r with $Y^+(r)$. When looking instead at individual snapshots, we find that the variations of these exponents range from ± 0.003 to ± 0.4, the larger for larger p. The dotted line represents the $p/3$ Kolmogorov scaling and the dashed line follows the SL model for fluids. Similarly, Fig. 6 gives, on the bottom, the scaling exponents ξ_p^- of the structure functions of the $\mathbf{z}^-$ field, namely $Y_p^-(r)\sim[Y^-(r)]^{\xi_p^-}$, with error bars varying from ± 0.001 to ± 0.1. Thus, one can observe that the fluid SL model applies for the Yaglom fields $Y_p^{\pm}$, as already obtained for Run A. Note that up to the maximum order shown in this figure, the log-normal model with its parameter μ set to the standard value $\mu=0.21$ would agree as well with the data. We thus confirm the result that, at least for two-dimensional flows, flux variables in MHD scale as neutral fluids, whereas physical fields are more intermittent since they display a stronger departure from linear scaling.

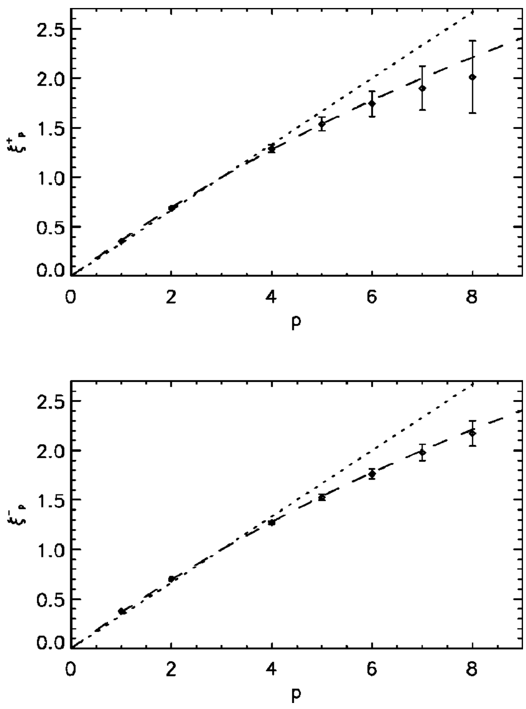

FIG. 6. Anomalous exponents $\xi_p^{\pm}$ [see relation (9)] with error bars, as a function of order p for Run D; also shown are the K41 law (dotted line) and the SL model for fluid flows (dashed line). On the top, ξ_p^+ is given with error bars ranging from ± 0.003 to ± 0.4; similarly on the bottom, ξ_p^- is given with error bars ranging from ± 0.001 to ± 0.1.

V. CONCLUSION

This paper has first briefly reviewed the theoretical modeling and the existing (mostly numerical) data concerning incompressible MHD turbulence which can be viewed as the result of a weakening of nonlinear transfer to small scales because of Alfvén wave propagation.[6,7] Such laws are an extension of the classical phenomenology for neutral fluids, dating back to von Kármán and Howarth,[43] and of course Kolmogorov. Such slowing down of nonlinear transfer in conducting flows is observed as well in the compressible case[44] and may help explain the ubiquity of supersonic flows in the interstellar medium, although an energy source is also required. Scaling laws for the energy, both temporal and spatial, are in agreement with the IK model[18,19] taking into account MHD effects, even though the anisotropy induced by the presence of a large scale or a uniform magnetic field[45] is not explicitly taken into account, but may only intervene in the case of weak MHD turbulence.[13] Similarly, the effect of back transfer in MHD (of the magnetic potential[46] in the two-dimensional case, and of the magnetic helicity in the three-dimensional case) is not taken into account either in such scaling laws, on the ground that inertial ranges, in sufficiently well-developed flows, are independent; hence, the drive for high-resolution runs.

When dealing with characteristic exponents of structure functions of higher order, the laws equivalent to the ''4/5'' law of Kolmogorov but derived in terms of the Elsässer variables[22,23] prove useful, in the framework of the ESS methodology, to delimit the inertial range within which scaling applies and thus to provide an independent variable for scaling.

Two conclusions arise from our analysis. On the one hand, the physical variables (velocity, magnetic field and their combination through the Elsässer variables) are noticeably more intermittent than in the fluid case. This is consistent with the Batchelor analogy between vorticity and magnetic field: there are magnetic flux tubes that are similar to vortex tubes (though not identical, since there is no constraint on the magnetic induction stemming from the Biot–Savart law). Hence, the magnetic field could be as intermittent as the vorticity itself. Indeed, it is known that strong magnetic flux tubes—of the order of a few kiloGauss, to be compared to a mean field of a Gauss—are observed indirectly on the solar surface, at the border of convective cells.

On the other hand, when dealing with the $\pm$ flux variables defined in (5), the anomalous exponents of a neutral fluid are recovered for these flux variables, indicating that such exponents may be a signature of advection and transport without being too sensitive to the details of the physical mechanisms by which the excitation is transferred to small scales. It should be noted that such identical exponents are obtained also in sustained supersonic flows with a r.m.s. Mach number of unity for a driving using three orthogonal shear waves,[41] as well as for the passive scalar,[38,39] lending some universality to turbulent flows when examined in the proper variables corresponding to the flux across scales. However, caution must be exercised insofar as only two-dimensional computations are performed in this paper, and the three-dimensional case remains to be done at high resolution. It would also be of great interest to pursue this investigation of anomalous exponents for other physical problems, such as they arise, e.g., in dispersive MHD.

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It is both a pleasure and an honor to participate in this special edition of Physics of Fluids dedicated to Professor R. H. Kraichnan for his seventieth birthday. We hope this paper shows how influential and relevant his ideas are to the understanding of complex MHD flows, as they are in many other fields as amply demonstrated by this issue. Bob, we wish you all the best!

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