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Dynamical length scales for turbulent magnetized flows

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Abstract. We derive two symmetric global scaling laws for third-order structure functions of magnetized fluids under the assumptions of full isotropy, homogeneity and incompressibility. The compatibility with previous laws involving both structure and correlation functions of only the longitudinal components of the fields is demonstrated. These new laws provide a better set of functions with which one can determine intermittency scaling of MHD turbulence, as in the Solar Wind.

Introduction

There are few exact results in turbulence (for a recent introduction, see *Gallavotti* [1993]), and one relies mostly on phenomenology to describe the statistical properties of turbulent flows. With the added complexity of coupling the velocity $\mathbf{v}$ to other variables such as the magnetic field $\mathbf{b}$, several phenomenological models have been developed which can only be sorted out by careful examination of data – experimental, observational and numerical. In the presence of a strong uniform magnetic field like in the Solar Wind, the equations can be linearized, showing that Alfvén and magnetosonic waves propagate in the medium; these waves interact, and sharp fronts develop in the form of magnetic flux tubes, vortex sheets and shocks. Such structures, small in at least one dimension, of finite extent in at least one transverse direction, are both strong and rare, characteristic of an intermittent fluid. There is ample evidence of turbulence [e.g., *Matthaeus and Goldstein,* 1982] and intermittency in the Solar Wind [*Burlaga,* 1991, 1993; Grappin et al., 1991; *Marsch and Liu,* 1993; *Marsch and Tu,* 1994; Ruzmaikin et al., 1995; Tu et al., 1996], although the turbulence may not always be fully developed, in particular close to the sun [*Tu and Marsch,* 1995]. Intermittency is diagnosed for example by examining the wings of probability distribution functions, wider than for a Gaussian with the same first two moments. Both laboratory and numerical data on neutral fluids show that these wings are exponentials; in the Solar Wind, there are also indications of power–law wings linked to the presence of Lévy laws [*Collier,* 1993]. Intermittency can be quantified, e.g., with longitudinal structure functions of order p defined as $S_p(\mathbf{r}) \equiv \langle(\delta v_L(\mathbf{r}))^p\rangle$ with $\delta v_L(\mathbf{r}) \equiv [\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})]\cdot\mathbf{r}$, where $\mathbf{r}$ is the displacement and brackets indicate space–averaging (hereafter, the subscript L denotes longitudinal components). Assuming homogeneity, incompressibility and full isotropy (without isotropy, see, e.g., *Frisch* [1995]), *Kolmogorov* [1941] derived the rigorous result which for $r \to 0$ in the inertial range reads:

$$S_3(\mathbf{r}) = -\frac{4}{5}\epsilon^V r \tag{1}$$

where $r = |\mathbf{r}|$ and $\epsilon^V \equiv -\partial_t E^V$ is the mean rate of kinetic energy transfer (a similar relationship holds for pressure [*Lindborg,* 1996]); viscous and forcing terms have been neglected in order to stress the behavior in the inertial domain, which can be defined as the range of scales $l_D < \ell < l_0$ with l_0 the integral scale characteristic of the large–scale eddies, and $l_D \sim (\epsilon^V/\nu^3)^{1/4}$ the dissipative length where ν is the viscosity. Conversely, when dealing with observational data, one can delimit empirically the inertial domain as the range of scales where (1) holds. Assuming that in the inertial range $S_p(\mathbf{r}) \sim r^{\zeta_p}$, an extension of the Kolmogorov phenomenology to all orders leads to $\zeta_p = p/3$; however, it does not take into account intermittency which gives strong corrections [*Sreenivasan and Antonia,* 1997]. Such corrections are incorporated in the scaling exponents as $\zeta_p = p/3 + \tau_{p/3}$ with obviously $\tau_0 = 0$ and $\tau_1 = 0$.

Numerous models attempt at predicting the behavior of $\tau_p = f(p)$ for turbulent fluids. They all differ at high p, for which the data is poor since it involves the rare events which dominate the statistics as p grows; thus, it is essential to examine the largest data sets at the largest Reynolds numbers available. Moreover, a better determination of scaling exponents obtains when S_p is expressed in terms of S_q with $q \neq p$ [Benzi et al., 1993]. This is interpreted as the extension of the self–similarity of turbulent flows beyond the inertial range defined by (1). Thus S_3, proportional to r according to equation (1), is a natural candidate for a "dynamical length" to study the scaling behavior of fluids. Data for Solar Wind turbulence can be calibrated so that $\zeta_3 = 1$; but one could also use the rigorous law equivalent to (1) for incompressible magnetized fluids obtained in *Politano and Pouquet* [1997], hereafter referred to as Paper I. However, without further assumptions on the amount of correlations between the velocity and the magnetic fields, nor on the relative intensity of these two fields,

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the two parameters that are known to determine the possible MHD regimes in the selective decay framework [Ting et al., 1983], the scaling laws of Paper I in MHD involve both third–order structure functions and correlation functions of the longitudinal fields. They are thus of a more complex nature than for neutral fluids. It is the purpose of this paper to derive new MHD scaling laws that are simpler than those obtained in Paper I, using a different approach now involving all the components of the basic fields. They can be used to examine data sets for MHD turbulence, as those from Voyager, Helios or Ulysses, and compute more accurately anomalous scaling exponents of structure functions of high order, in the spirit of Benzi et al. [1993].

New Scaling Laws for MHD

The analysis is best performed using the symmetry of the incompressible MHD equations in terms of the Elsässer fields $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ with $\nabla \cdot \mathbf{z}^{\pm} = 0$:

$$\partial_t \mathbf{z}^{\pm} = -(\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} - \nabla P_* + \nu \nabla^2 \mathbf{z}^{\pm} \qquad (2)$$

where $P_* = P + b^2/2$ is the total pressure. For simplicity, we take a unit magnetic Prandtl number P^M (recall that, using the Renormalization Group formalism, Fournier et al. [1982] have shown that the turbulent viscosity and magnetic diffusivity settle at a ratio close to unity). The pressure term poses no particular problem and is omitted from now on, since in the analysis it gives a null contribution due to incompressibility and space averaging.

We now derive an equation for the evolution of the second order structure function of the $\mathbf{z}^{\pm}$ fields which leads to scaling equations for $\langle \delta z_L^{\mp}(\mathbf{r})(\delta z_i^{\pm}(\mathbf{r}))^2\rangle$ (with summation over repeated indices understood). As noted [Antonia et al., 1997; *Fulachier and Dumas,* 1976] for the fluid case, this scaling law is similar to that found by *Yaglom* [1949] for the scaling of the temperature within a turbulent flow (see Caillol et al. [1996] for the case of the magnetic potential in 2D MHD). The same type of law holds as well for the direct energy cascade in MHD, both in two and three dimensions. Writing (2) at $\mathbf{x}' = \mathbf{x} + \mathbf{r}$, substracting these two equations and multiplying the result by $\delta z_i^{\pm}(\mathbf{r})$, one obtains the temporal evolution of $\langle (\delta z_i^{\pm}(\mathbf{r}))^2\rangle = \langle (z_i^{\pm}(\mathbf{x}') - z_i^{\pm}(\mathbf{x}))^2\rangle$. Using homogeneity, incompressibility and the independence of $\mathbf{x}$ and $\mathbf{x}'$ yields:

$$\begin{aligned}\partial_t \langle (\delta z_i^{\pm}(\mathbf{r}))^2\rangle &= -\partial_{r_k}\langle \delta z_k^{\mp}(\mathbf{r})(\delta z_i^{\pm}(\mathbf{r}))^2\rangle \\ &\quad +2\nu \partial^2_{r_k}\langle (\delta z_i^{\pm}(\mathbf{r}))^2\rangle \\ &\quad -4\nu \langle (\partial_{x_k} z_i^{\pm}(\mathbf{x}))^2\rangle, \qquad (3)\end{aligned}$$

(see also [*Marsch and Tu,* 1989; *Zhou and Matthaeus,* 1989] for evolution equations for $\langle {z_i^{\pm}}^2\rangle$). The third term on the *r.h.s* of (3) can be identified with the mean energy transfer and dissipation rates $\epsilon^+ \equiv -\partial_t E^+ = \nu\langle (\partial_{x_k} z_i^+(\mathbf{x}))^2\rangle = \partial_{r_k}(\epsilon^+ r_k)/d$, where d is the space dimension. When now isotropy is assumed, the tensor inside the derivative of the nonlinear term of the *r.h.s* of (3) is a first–order tensor, thus proportional to r_k, and one easily obtains in the inertial range:

$$\langle \delta z_L^-(\mathbf{r})(\delta z_i^+(\mathbf{r}))^2\rangle = -\frac{4}{d}\epsilon^+ \, r; \qquad (4)$$

similarly, for the $\mathbf{z}^-$ field, where ϵ^- stands for the energy transfer and dissipation rates of the $\mathbf{z}^-$ variable:

$$\langle \delta z_L^+(\mathbf{r})(\delta z_i^-(\mathbf{r}))^2\rangle = -\frac{4}{d}\epsilon^- \, r. \qquad (5)$$

Equations (4) and (5) are the main results of this paper. Returning to the basic fields one equivalently obtains, with the r–dependence in the *l.h.s.* omitted:

$$\langle \delta v_L(\delta v_i)^2\rangle + \langle \delta v_L(\delta b_i)^2\rangle - 2\langle \delta b_L \delta v_i \delta b_i\rangle = -\frac{4}{d}\epsilon^T \, r \quad (6)$$

$$-\langle \delta b_L(\delta b_i)^2\rangle - \langle \delta b_L(\delta v_i)^2\rangle + 2\langle \delta v_L \delta v_i \delta b_i\rangle = -\frac{4}{d}\epsilon^C \, r \quad (7)$$

with $\epsilon^T = (\epsilon^+ + \epsilon^-)/2$ and $\epsilon^C = (\epsilon^+ - \epsilon^-)/2$ the transfer and dissipation rates of the total energy $\langle (v_i^2 + b_i^2)/2\rangle$ and the correlation $\langle \mathbf{v} \cdot \mathbf{b}\rangle$. For $\mathbf{b} \equiv 0$, one recovers the previously known relationship for neutral fluids (*op. cit.*). Note that in the case of magnetically dominated flows, it is now the third–order structure functions $\langle \delta b_L(\delta b_i)^2\rangle$ that scales as r, granted the velocity–magnetic field correlations are non–zero (even if weak). In fact, it can be conjectured that all the group s of terms in (6), and independently in (7), which are of the same order in $\langle b_i^2\rangle / \langle v_i^2\rangle$, scale individually as r.

On the other hand one can derive (see Paper I), starting again from equation (2) and following *von Kármán–Howarth* [1938] and *Kolmogorov* [1941], the relationships for MHD similar to the Kolmogorov law and involving only the longitudinal components of the fields:

$$\langle \delta z_L^{\mp}(\mathbf{r})(\delta z_L^{\pm}(\mathbf{r}))^2\rangle - 2\langle z_L^{\pm}(\mathbf{x}) z_L^{\pm}(\mathbf{x}) z_L^{\mp}(\mathbf{x}')\rangle = -C_d \epsilon^{\pm} r \quad (8)$$

with $C_d = 8/(d(d+2))$. Preliminary numerical simulations in two dimensions confirm these scalings [Pouquet et al., 1997]. As opposed to equations (4) and (5), these laws (i) involve *both* correlation and structure functions, and are thus more complex and (ii) they only include the longitudinal components of the fields. Moreover, their derivations are quite different; indeed, in the approach of Paper I, ϵ^+ is defined as $\epsilon^+ = -d\,\partial_t\langle {z_L^+}^2\rangle/2$ (and similarly for ϵ^-) using isotropy, whereas here it is defined from the dissipative terms. Furthermore, the laws in Paper I obtain independently of the value of the magnetic Prandtl number (although P^M enters the analysis through the hypothesis of the existence and of the extent in scales of the inertial range [*Kraichnan and Nagarajan,* 1967] within which they apply.

Compatibility of the two MHD Laws

For three–dimensional neutral flows, the compatibility between equation (1) and the law $\langle \delta v_L(\mathbf{r})(\delta v_i(\mathbf{r}))^2\rangle = -(4/3)\epsilon^V r$ obtains immediately. We define the third order structure function $B_{ijl}(\mathbf{r}) \equiv \langle \delta v_i(\mathbf{r})\delta v_j(\mathbf{r})\delta v_l(\mathbf{r})\rangle$. The longitudinal structure and correlation functions of the velocity are proportional in the fluid case [*Batchelor,* 1953]. ¿From this, one obtains a simple relationship between the longitudinal structure function and the one involving the transverse components, namely $6B_{Lnn}(\mathbf{r}) = \partial_r(rB_{LLL}(\mathbf{r}))$ where the n subscript refers to any of the two normal components of the field with respect to the $\mathbf{r}$–direction. This leads to the desired equivalence between the two formulations.

In MHD, however, the derivation of the compatibility between equations (4)–(5) and (8) is slightly more involved. We first define the third–order correlation tensor $C_{ijl}^{ss's''}(\mathbf{r}) \equiv \langle z_i^s(\mathbf{x})z_j^{s'}(\mathbf{x})z_l^{s''}(\mathbf{x}')\rangle$ with s, s' and s'' equal to $\pm$, and the third–order structure function $B_{ijl}^{ss's''}(\mathbf{r}) \equiv \langle \delta z_i^s(\mathbf{r})\delta z_j^{s'}(\mathbf{r})\delta z_l^{s''}(\mathbf{r})\rangle$. One notes that

$$B_{LLL}^{-++} = 2C_{LLL}^{++-} + 4C_{LLL}^{-++} \qquad (9)$$

(omitting the $\mathbf{r}$–dependence from now on) and also $B_{Lnn}^{-++} = 2C_{nnL}^{++-} + 4C_{Lnn}^{-++}$. As is well-known [*Batchelor,* 1953], using incompressibility, any third-order correlation tensors (with both the super- and sub-scripts fixed) can be expressed in terms of only one defining function related to the longitudinal correlation functions, viz. $C_3^{ss's''}k^{ss's''}(r) \equiv C_{LLL}^{ss's''}$ with $C_3^{ss's''} = z^s z^{s'} z^{s''}$ where $z^\pm$ stand for the *r.m.s.* $\mathbf{z}^\pm$ fields. Hence: $B_{Lnn}^{-++} = \frac{2C_3^{-++}}{d-1}\left((d-1)k^{-++} + r\partial_r k^{-++} - k^{++-}\right)$, and thus:

$$\begin{aligned}\langle \delta z_L^-(\delta z_i^+)^2\rangle &= B_{LLL}^{-++} + (d-1)B_{Lnn}^{-++} \\ &= \frac{2}{r^d}\partial_r\left(r^{d+1}C_{LLL}^{-++}\right). \qquad (10)\end{aligned}$$

Using (8) and (9) leads to $C_{LLL}^{-++} = -C_d\,\epsilon^+ r/4$. Finally, expressing the derivative in (10), one arrives at equation (4). Equation (5) recovers in a similar way. Hence in MHD as well, the scaling laws for longitudinal correlation and structure functions and those for scalars derived in this paper for $P^M = 1$ are compatible.

Discussion

The results given here provide a better theoretical understanding per se of incompressible MHD flows. They may yield as well an improved basis for the calibration of data sets of MHD turbulence, including for the Solar Wind, although in that latter case the assumptions of incompressibility and full isotropy and discarding kinetic effects are somewhat unrealistic [*Tu and Marsch,* 1995], yet essential in the analysis in more than one space dimension. Note that incorporating kinetic effects may lead to MHD–like behavior, e.g., [*Strauss,* 1991; *Hada,* 1993]. In the case of strong magnetic fields, one has simply $\langle \delta b_L(\mathbf{r})(\delta b_i(\mathbf{r}))^2\rangle \sim r$, and $\langle(\delta b_L(\mathbf{r}))^3\rangle \sim r$ as well, which may also hold in the compressible case since the velocity is not involved (the equivalent scaling laws for 1D compressible MHD fluids are derived in Galtier et al. [1997]). The *Iroshnikov* [1963] and *Kraichnan* [1965] phenomenology predicts that $\zeta_2^\pm = 1/2$, where $\zeta_2^\pm$ are the scaling exponents of the second–order structure functions of $\mathbf{z}^\pm$; it relies on taking into account the slowing–down of the energy transfer due to the interaction of $\mathbf{z}^\pm$ eddies propagating in opposite directions along a uniform large–scale magnetic fied B_0. An essential assumption behind it, is that the correlations $\langle \mathbf{v}\cdot\mathbf{b}\rangle$ are negligible. For strongly correlated flows, one simply finds on heuristic grounds [Grappin et al., 1983] that $\zeta_2^+ + \zeta_2^- = 1$, leaving the individual exponents of the $\pm$ spectra undetermined. The present paper is free from such an assumption. It provides an exact result at the level of third–order structure functions for any amount of $(\mathbf{v}, \mathbf{b})$ correlation. It also indicates that correlations between the $\mathbf{z}^\pm$ fields are of course an essential part of the nonlinear dynamics in MHD. One may be in a regime where $v \sim b$ in amplitude, but this does not imply that $\langle(\delta v_L(\mathbf{r}))^3\rangle$ scales as $\langle(\delta b_L(\mathbf{r}))^3\rangle$; in the opposite regime where $z^+ \sim z^-$, similarly, one cannot infer either that $\langle(\delta z_L^+(\mathbf{r}))^3\rangle$ scales as $\langle(\delta z_L^-(\mathbf{r}))^3\rangle$. Indeed, there exist non–zero spatial correlations between the basic fields, correlations implied by the conservation properties of the primitive equations and correlations that affect the scaling laws leading to a scaling different from that of *Kolmogorov* [1941] for neutral fluids. Hence, as shown in this paper, the third–order structure functions $\langle \delta z_L^\mp(\mathbf{r})\,(\delta z_i^\pm(\mathbf{r}))^2\rangle$ may well provide a better set of variables (compared to r itself, or to $\langle(\delta v_L(\mathbf{r}))^3\rangle$) proportional to length against which to fit the data, including beyond the inertial range as in the fluid case; this will lead to a better evaluation of anomalous exponents of structure functions. They should therefore be used in analyzing high–order structure functions of various data sets for conducting flows, in particular for the Solar Wind, thus shedding a needed light on the intermittency properties of MHD–like turbulent flows, and as well on the extent of the validity of the laws proposed here.

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