Université de Nice-Sophia Antipolis
L3 MIAGE - Processus stochastiques et analyse de données
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Examen, corrigé du sujet B

Durée : 1h. Documents, calculatrices et téléphones interdits.

1. Puisque X et Y sont indépendants : $\mathbb{P}(\{X \leq 1/2\} \cap \{Y \leq 1/3\}) = \mathbb{P}(X \leq 1/2)\mathbb{P}(Y \leq 1/3) = (1/2)(1/3) = 1/6$.
2.
   (a) $\mathbb{E}(U) = \int_1^3 \frac{1}{2} u du = [\frac{u^2}{4}]_1^3 = \frac{9-1}{4} = \frac{8}{4} = 2$
   (b) $\mathbb{E}(U^2) = \int_1^3 \frac{1}{2} u^2 du = [\frac{u^3}{6}]_1^3 = \frac{27-1}{6} = \frac{26}{6} = \frac{13}{3}$
   (c) $\mathrm{Var}(U) = \frac{13}{3} - 4 = \frac{13-12}{3} = \frac{1}{3}$
3. $\mathbb{P}(X \leq 2) = \mathbb{P}(\{X = 1\} \cup \{X = 2\}) = \mathbb{P}(X = 1) + \mathbb{P}(X = 2) = 2/6 = 1/3$
4. $C_3^2 (1/6)^2 (1 - 1/6) = 3 \frac{1}{6^2} \frac{5}{6} = \frac{5}{72}$
5. $\mathbb{P}(Z < -0,65) = \mathbb{P}(Z \geq 0,65) = 1 - \mathbb{P}(Z < 0,65) = 1 - 0,7422 = 0,2578$
6. $\mathbb{P}(\{X = 2\} \cup \{X = 1\}) = \mathbb{P}(X = 2) + \mathbb{P}(X = 1) = C_3^2 (1/3)^2 (1-1/3)^{3-2} + C_3^1 (1/3)^1 (1-1/3)^{3-1} = 3 \frac{1}{9} \frac{2}{3} + 3 \frac{1}{3} \frac{4}{9} = \frac{2}{3}$
7.
   (a) La variable Z peut prendre les valeurs : 2, 0, 3, 1.
   (b) $\mathbb{P}(Z = 2) = \mathbb{P}(X = 1, Y = 1) = (3/4)(1/3) = 1/4$, $\mathbb{P}(Z = 0) = \mathbb{P}(X = 1, Y = -1) = (3/4)(2/3) = 1/2$, $\mathbb{P}(Z = 3) = \mathbb{P}(X = 2, Y = 1) = (1/4)(1/3) = 1/12$, $\mathbb{P}(Z = 1) = \mathbb{P}(X = 2, Y = -1) = (1/4)(2/3) = 1/6$