Conference "SCV, CR geometry and Dynamics"
May 30th-June 3rd 2023, Nice

- **Masanori Adachi** (Shizuoka U.):"On weighted Bergman spaces of a domain with Levi-flat boundary"

  Abstract For each compact hyperbolic Riemann surface we may attach a canonical ruled surface over it using its uniformization. This ruled surface contains a Levi-flat real hypersurface that divides the surface into two 1-convex domains. A feature of this construction is that these two domains admit bounded psh exhaustions but no bounded holomorphic functions except for constants. We discuss structure theorems for the space of holomorphic functions on these domains.

- **Fabrizio Bianchi** (U. de Lille):"Every complex Hénon map satisfies the Central Limit Theorem"

  Abstract We show that the measure of maximal entropy of every complex Hénon map is exponentially mixing of all orders for Hölder observables. As a consequence, the Central Limit Theorem holds for all Hölder observables. A similar property holds for every automorphism of a compact Kähler manifold with simple action on cohomology. This is a joint work with Tien-Cuong Dinh.

- **Tien Cuong Dinh** (National U. of Singapore):" Properties of dynamical degrees"

  Abstract Dynamical degrees measure the growth of subvarieties and currents under the action of the dynamical system. They play a crucial role in complex dynamics, in particular in the construction of Green's dynamical currents, the construction of maximum entropy measures, equidistribution problems and many ergodic properties. In this talk, we will review some properties of dynamical degrees and a recent result on their monotonicity for Hénon-like and polynomial-like maps. This talk is based on my joint works with Fabrizio Bianchi, Viet-Anh Nguyen, Karim Rakhimov and Nessim Sibony.

- **Peter Ebenfelt** (UCSD):"Kähler geometry and obstruction flat CR manifolds"

  Abstract Let $X = X^{2n+1}$ be a C^∞-smooth strictly pseudoconvex CR hypersurface in a complex manifold of dimension $n+1$. There exists a local defining function ρ with a *finite* degree of smoothness, namely $C^{n+3-\epsilon}$, up to X such that $\omega = i\partial\bar\partial \log \rho$ defines a Kähler–Einstein metric on the pseudoconvex side of X. In general, higher degree smoothness of ρ up to X is obstructed by a log-term whose coefficient is a local CR invariant of X. The CR manifold X is said to be *obstruction flat* if this obstruction invariant vanishes. It is easy to see that if X is spherical, then it is obstruction flat. For $n = 1$, there is ample evidence that the converse holds, i.e., if $X = X^3$ is compact and obstruction flat, then it is spherical; this is sometimes referred to as the Strong Ramadanov Conjecture for reasons that will be explained in the talk. For $n \geq 2$, this is no longer true and the classification of compact, obstruction flat CR manifolds of higher dimension is wide open. In this talk,

we shall discuss the special case where the CR manifold X arises as the unit circle bundle of a negative Hermitian line bundle over a Kähler manifold.

- **Sairhei Finski** (E. Polytechnique):"Geometry at the infinity of the space of positive metrics"

  Abstract From the work of Phong and Sturm in 2007, for a polarised projective manifold and a test configuration (which is some special degeneration of the manifold), one can associate a ray of positive metrics on the polarising line bundle using the solution of the Monge-Ampère equation on the resolution of singularities of the test configuration. We prove that the Mabuchi chordal distance between the rays associated with two test configurations can be calculated algebraically through the non-Archimedean distance between the associated filtrations on the section ring. This result should be interpreted as an algebraic description of the boundary at infinity of the space of positive metrics, viewed — as it is usually done for spaces of negative curvature — in terms of geodesic rays.

- **Xianghong Gong** (U. Wisconsin-Madison):" A structure theorem for neighborhoods of compact complex manifolds"

  Abstract One of main results in the holomorphic equivalence of neighborhoods M of a compact complex manifold C is Grauert's formal principle, which asserts that two holomorphic neighborhoods are holomorphically equivalent if they are formally equivalent. The formal principle was known to be true when the normal bundle of C in M is either weakly negative or 1-positive in the sense of Grauert. We will construct an injective map from the set of holomorphic equivalence classes of neighborhoods M into a certain finite-dimensional cohomology group when the normal bundle is either weakly negative or 2-positive. This yields a certain finite-dimensionality on the equivalence classes of neighborhoods, which is a new type of results. It was known previously that the finite dimensionality fails for some neighborhoods with 1-positive normal bundle. This is joint work with Laurent Stolovitch.

- **Purvi Gupta** (Indian Institute of Science):"Polyhedral-like approximations of strongly $\mathbb{C}$-convex domains"

  Abstract Polyhedral approximations of convex bodies have earned much attention in both affine and stochastic geometry. Of particular interest is the asymptotic analysis of the approximation error measured as a function of the complexity of the approximating polyhedral. This analysis yields invariant combinatorial and geometric data associated to the underlying convex body. In this talk, we will discuss the motivation to study polyhedral-like approximations of domains satisfying complex notions of convexity. In particular, we will focus on the notion of $\mathbb{C}$-convexity, which is a natural analogue of convexity in complex projective spaces. We will introduce a suitable class of polyhedra in this context and present some asymptotic results in the spirit of several results in (real) convex geometry.

- **Chin-Yu Hsiao** (Academia Sinica, Taipei):"Bergman kernel asymptotics on weakly pseudoconvex domains"

Abstract In this talk, I will report my recent work about Bergman kernel asymptotics on weakly pseudoconvex domains. This talk is based on joint work with George Marinescu, Xiaoshan Li and Nikhil Savale.

- **Xiaojun Huang** (Rutgers U.):" Curvature property of Bergman metrics over a complex manifold"

Abstract I will discuss the curvature property of Bergman metrics over a complex manifold. For the theorem I will discuss is when a pesudoconvex domain has constant holomorphic sectional curvature. We also present some examples to demonstrate the Bergman metric can take positive constant or can be flat along a totally geodesic submanifold. This will be a joint work with Song-Ying Li from UC Irvine.

- **Jun-Muk Hwang** (Institutue for Basic Science, Korea):"Torsion tensors of connections for isotrivial cone structures"

Abstract A cone structure on a complex manifold M is a submanifold in the projectivized tangent bundle of M which is submersive over M. Many interesting examples arise from minimal rational curves in algebraic geometry. When a cone structure is isotrivial, namely, when its fibers have a fixed isomorphism type, it is naturally associated to a G-structure where G is the linear automorphism group of the fiber. We discuss the relation between conic connections on isotrivial cone structures and principal connections on the associated G-structures, especially the relation between their torion tensors. For a certain class of G-structures, this can be used to classify possible cone structures arising from minimal rational curves.

- **Sung-Yeon Kim** (Institutue for Basic Science, Korea):"Proper holomorphic maps between bounded symmetric domains with small rank differences"

Abstract In this talk, we study the rigidity of proper holomorphic maps $f\colon \Omega \to \Omega'$ between irreducible bounded symmetric domains Ω and Ω' with small rank differences: $2 \leq \operatorname{rank}(\Omega') < 2\operatorname{rank}(\Omega) - 1$. More precisely, if either Ω and Ω' have the same type or Ω is of type III and Ω' is of type I, then up to automorphisms, f is of the form $f = \imath \circ F$, where $F = F_1 \times F_2 \colon \Omega \to \Omega'_1 \times \Omega'_2$. Here Ω'_1, Ω'_2 are bounded symmetric domains, the map $F_1 \colon \Omega \to \Omega'_1$ is a standard embedding, $F_2 : \Omega \to \Omega'_2$, and $\imath \colon \Omega'_1 \times \Omega'_2 \to \Omega'$ is a totally geodesic holomorphic isometric embedding. As a consequence, $f : \Omega \to \Omega'$ is a holomorphic totally geodesic isometric embedding with respect to Kobayashi metrics. Moreover we show that, under the rank condition above, there exists no proper holomorphic map $f : \Omega \to \Omega'$ if Ω is of type I and Ω' is of type III, or Ω is of type II and Ω' is either of type I or III. This is a joint work with N. Mok and A. Seo.

- **Takayuki Koike** (Osaka Metropolitan U.):"Holomorphic foliation associated with a semi-positive class of numerical dimension one"

Abstract Let X be a compact Kähler manifold and α be a class in the Dolbeault cohomology class of bidegree $(1,1)$ on X. When the numerical dimension of α is one and α admits at least two smooth semi-positive representatives, we show the existence of a

family of real analytic Levi-flat hypersurfaces in X and a holomorphic foliation on a suitable domain of X along whose leaves any semi-positive representative of α is zero. As an application, we give the affirmative answer to a conjecture on the relation between the semi-positivity of the line bundle $[Y]$ and the analytic structure of a neighborhood of Y for a smooth connected hypersurface Y of X.

- **Ilya Kossovskiy** (Marasyk U.):"Fuchsian CR manifolds and applications"

  Abstract In this talk, I will outline the notion of Fuchsianity in CR-geometry, its developments and applications for studying holomorphic extension properties of CR-maps.

- **Bernhard Lamel** (Texas A&M at Qatar):"Realization of formal CR automorphisms"

  Abstract We discuss joint work with Kossovskiy and Stolovitch showing that every formal holomorphic automorphism of a real-analytic hypersurface in $\mathbb{C}^2$ is actually the Taylor series of a smooth CR diffeomorphism.

- **George Marinescu** (U. Köln):"Bochner-Laplacian and Bergman kernel expansion "

  Abstract Using a local index theory method going back to Bismut-Lebeau, we developed together with Dai, Liu, and Ma a method for proving the expansion of the Bergman kernel of high tensor powers of a positive line bundle. I will present a joint result with Savale on the asymptotics of the first eigenvalue, and in particular a spectral gap, for the Bochner Laplacian acting on tensor powers while allowing the curvature to vanish. An application is the Bergman kernel expansion at vanishing points of the curvature for semi-positive line bundles on a Riemann surface. Time permitting, I will discuss joint results with Xiaonan Ma and Nikhil Savale in the setting of families.

- **Nordine Mir** (Texas A&M at Qatar):"Unique jet determination of CR maps into Nash manifolds"

  Abstract We present recent results about finite jet determination of CR maps of positive codimension from real-analytic CR manifolds into Nash manifolds (or sets) in complex space. One instance of such results is the unique jet determination of germs of CR maps from minimal real-analytic CR submanifolds in $\mathbb{C}^N$ into Nash subsets in $\mathbb{C}^{N'}$ of D'Angelo finite type, for arbitrary $N, N' \geq 2$. This is joint work with B. Lamel and G. Rond.

- **Ngaiming Mok** (Hong-Kong U.):"Abelian schemes over complex function fields and functional transcendence results"

  Abstract The speaker has long been interested in applications of complex geometry to number theory, and will trace the trajectory of his involvement revolving around abelian schemes over complex function fields and functional transcendence results on Shimura varieties with applications.

  We recall first results of Mok (1991) and Mok-To (1993) concerning the finiteness of Mordell-Weil groups of universal abelian varieties $\mathbf{A}_\Gamma$ without fixed parts over modular function fields $K = \mathbb{C}(\overline{X_\Gamma})$. In these early works in the modular case an invariant Kähler

form (currently known as the Betti form) due to Satake was introduced, and, making use of the classifying map, ranks of Mordell-Weil groups were bounded in geometric terms via the volume of the ramification locus. An important tool was the extendibility of the Betti form as a closed positive current, established in the above works as an intermediate tool using the methods of P. Lelong and H. Skoda on closed positive currents.

Most recently, Mok-Ng (2022) applied the complex differential geometric approach in the above to prove finiteness results on points of Betti multiplicities ≥ 2 of a section $\sigma \in \mathbf{E}(\mathbb{C}(\overline{X}))$ of infinite order in the case of an elliptic scheme over a quasi-projective curve, a result obtained by Corvaja-Demeio-Masser-Zannier, which was rendered effective by Ulmer-Urzúa (2021). Our approach has the advantage of being applicable in principle to abelian schemes.

Regarding functional transcendence results we will discuss the Ax-Schanuel theorem of Mok-Pila-Tsimerman (2019) for Shimura varieties and applications of its generalizations to the study of rational points, notably to the proof of the uniform Mordell-Lang theorem of Dimitrov-Gao-Habegger (2021) for number fields, and the characterization of bi-algebraicity due to Chan-Mok (2022) in the case of a projective subvariety $Y \subset X_{\check{\Gamma}}$, for $X_{\check{\Gamma}}$ possibly of infinite volume, uniformized by an algebraic subset $Z \subset \Omega$.

- **Duong Phong** (Colombia U.):"Symplectic geometric flows and almost-complex structures"

  Abstract The equations of unified string theories have led to several new interesting geometric flows. One of which is the Type IIA flow, which is a weakly parabolic flow on 6d symplectic manifolds whose underlying geometry turns out to be SU(3) holonomy, but for a *projected* Levi-Civita connection with respect to an almost-complex structure. We discuss this flow and many open related flows in symplectic geometry. This is joint work with Teng Fei, Sebastien Picard, and Xiangwen Zhang.

- **Dan Popovici** (U. de Toulouse):"Partially Hyperbolic Compact Complex Manifolds"

  Abstract This is joint work with H. Kasuya. We propose and investigate two types, the latter with two variants, of notions of partial hyperbolicity accounting for several classes of compact complex manifolds behaving hyperbolically in certain directions, defined by a vector subbundle of the holomorphic tangent bundle, but not necessarily in the other directions. We study several classes of examples, prove implications among these notions, give a sufficient criterion for the existence of an Ahlfors current and a sufficient criterion for partial hyperbolicity in terms of the signs of two curvature-like objects introduced recently by the second-named author.

- **Nikhil Savale** (U. Köln):"Kahler-Einstein Bergman metrics on pseudoconvex domains of dimension two"

  Abstract This talk consists of two parts. In the first part, I will review the construction of a pointwise Boutet de Monvel-Sjostrand parametrix for the Szego kernel of a weakly pseudoconvex three dimensional CR manifold of finite type and Fefferman type boundary asymptotics of the Bergman kernel on weakly pseudoconvex domains in dimension two. The second part presents an application where we prove that a weakly pseudoconvex two

dimensional domain of finite type with a Kahler-Einstein Bergman metric is biholomorphic to the unit ball. Based on joint works with C.Y. Hsiao and M. Xiao.

- **Ziming Shi** (UC-Irvine):"$\frac{1}{2}$-Estimate for global Newlander-Nirenberg theorem on strongly pseudoconvex domains"

  Abstract Given a formally integrable almost complex structure X defined on the closure of a bounded domain $D \subset \mathbb{C}^n$, and provided that X is sufficiently close to the standard complex structure, the global Newlander-Nirenberg problem asks whether there exists a global diffeomorphism defined on $\overline{D}$ that transforms X into the standard complex structure, under certain geometric and regularity assumptions on D. In this paper we prove a quantitative result of this problem. Assuming D is a strongly pseudoconvex domain in $\mathbb{C}^n$ with C^2 boundary, and that the almost structure X is of the Hölder-Zygmund class $\Lambda^r(\overline{D})$ for $r > \frac{3}{2}$, we prove the existence of a global diffeomorphism (independent of r) in the class $\Lambda^{r+\frac{1}{2}-\varepsilon}(\overline{D})$, for any $\varepsilon > 0$. The main idea in our proof is the construction of a family of Moser type smoothing operators on bounded Lipschitz domains.

- **Duc-Viet Vu** (Köln U.):"Bergman kernel functions associated to measures supported on totally real submanifolds"

  Abstract In the talk I will explain my recent joint-work with George Marinescu (Cologne) in which we prove that the Bergman kernel function associated to a smooth measure supported on a piecewise-smooth maximally totally real submanifold K in $\mathbb{C}^n$ is of polynomial growth (e.g, in dimension one, K is a finite union of transverse Jordan arcs in $\mathbb{C}$). Our bounds are sharp when K is smooth. We also discuss an application to equidistribution of zeros of random polynomials extending a result of Shiffman-Zelditch to the higher dimensional setting.

- **Dmitri Zaitsev** (Trinity Colllege):"Catlin's global regularity - a new proof and generalizations"

  Abstract The celebrated result of Catlin on global regularity of the $\bar{\partial}$-Neumann operator for pseudoconvex domains of finite type links local algebraic- and analytic geometric invariants through potential theory with estimates for $\bar{\partial}$-equation. Yet despite their importance, there seems to be a major lack of understanding of Catlin's techniques, resulting in a notable absence of an alternative proof, exposition or simplification.

  The goal of my talk will be to present an alternative proof based on a new notion of a "tower multi-type". The finiteness of the tower multi-type is an intrinsic geometric condition that is more general than the finiteness of the regular type, which in turn is more general than the finite type. Under that condition, we obtain a "stratification" of the boundary into countably many level sets of the tower multi-type, each covered locally by strongly pseudoconvex submanifolds of the boundary. The existence of such stratification implies Catlin's celebrated potential-theoretic "Property (P)", which, in turn, is known to imply global regularity via compactness estimate. Notable applications of global regularity include Condition R and applications to boundary smoothness of proper holomorphic maps.