

# Journée Complexe

Salle I, 23 Septembre 2022

9h30 **Junyan Cao** (Université Côte d'Azur): "On extension of pluricanonical bundle of a Kähler family"

Abstract: In this talk, I will explain a few results related to Y.-T. Siu's conjecture of deformational invariance of plurigenera for Kähler families and also some applications in algebraic geometry. It is a joint work with M. Paun.

11:00 **Masanori Adachi** (Shizuoka University): "On weighted Bergman spaces of a domain with Levi-flat boundary"

Abstract: For each compact hyperbolic Riemann surface we may attach a canonical ruled surface over it using its uniformization. This ruled surface contains a Levi-flat real hypersurface that divides the surface into two 1-convex domains. A feature of this construction is that these two domains admit bounded psh exhaustions but no bounded holomorphic functions except for constants, and these properties have roots in the ergodicity of the geodesic flow on the hyperbolic surface. I would like to give an overview around this example, in particular, to explain structure theorems for the space of holomorphic functions on these domains.

14:00 **Stéphanie Nivoche**(Université Côte d'Azur): "New solution of a problem of Kolmogorov on width asymptotics in holomorphic function spaces"

Abstract: Given a domain  $D$  in  $\mathbb{C}^n$  and  $K$  a compact subset of  $D$ , we denote  $\mathcal{A}_K^D$  the compact set in  $C(K)$ , of all restrictions in  $K$  of holomorphic functions on  $D$  bounded by 1. The sequence  $(d_m(\mathcal{A}_K^D))_{m \in \mathbb{N}}$  of Kolmogorov  $m$ -widths of  $\mathcal{A}_K^D$  provides a measure of the degree of compactness of the set  $\mathcal{A}_K^D$  in  $C(K)$  and the study of its asymptotics has a long history, essentially going back to Kolmogorov's work on  $\epsilon$ -entropy of compact sets in the 1950s. The precise statement of this problem is

$$\lim_{m \rightarrow \infty} \frac{-\log d_m(\mathcal{A}_K^D)}{m^{1/n}} = 2\pi \left( \frac{n!}{C(K, D)} \right)^{1/n}, \quad (1)$$

where  $C(K, D)$  is the Bedford-Taylor relative capacity of  $K$  in  $D$ . This problem has already been proved in 2004 by S.N., using pluripotential theory technics.

Here, with O. Bandtlow, we give a totally new proof of the asymptotics (1) for  $D$  strictly hyperconvex and  $K$  non-pluripolar. We proceed by a two-pronged approach establishing sharp upper and lower bounds for the Kolmogorov widths. The lower bounds follow from concentration results for the eigenvalues of a certain family of Toeplitz operators, while the

upper bounds follow from an application of the Bergman-Weil formula together with an exhaustion procedure by special holomorphic polyhedra.

15:30 **Takayuki Koike** (Osaka Metropolitan University): "A gluing construction of projective K3 surfaces"

Abstract: We construct a non-Kummer projective K3 surface  $X$  by holomorphically patching two open complex surfaces obtained as the complements of tubular neighborhoods of elliptic curves embedded in blow-ups of the projective plane at nine general points. Our  $X$  admits an open domain  $V \subset X$  which is holomorphically foliated by Riemann surfaces, and also real-analytically by compact Levi-flats. This talk is based on a collaboration with Takato Uehara at Okayama University.