Presenting de Groot duality of stably compact spaces by entailment relations

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(Logical approach to de Groot duality)

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De Groot duality of stably compact spaces

A topological space is **stably compact** if it is sober, locally compact, and finite intersections of compact saturated subsets are compact.

- Compact Hausdorff spaces.
- Scott topologies of continuous domains.
- Spectral spaces

De Groot dual X^{d} of a stably compact space X is a set X with the topology generated from complements of compact saturated subsets. The space X^{d} is stably compact and $(X^{\mathsf{d}})^{\mathsf{d}} = X$.

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Example

1. **(Lower vs. Upper)** Lower and upper Dedekind cuts of $[0, 1]$.
2. **(Open vs. Closed)** Scott topology $\Sigma(X)$ and Lower powerdomain $\mathrm{P_L}(X)$: $\Sigma(X)^{\mathsf{d}} \cong \mathrm{P_L}(X)$.
3. **(Closed vs. Compact)** Lower powerdomain $\mathrm{P_L}(X)$ and Upper powerdomain $\mathrm{P_U}(X)$: $\mathrm{P_L}(X^{\mathsf{d}}) \cong \mathrm{P_U}(X)^{\mathsf{d}}$.

Aim

Give a pointfree account of de Groot duality for stably compact locales.

“How to **present** de Groot duality”

Stably compact locales

Definition

A locale X is **spectral** if it is the ideals of a distributive lattice.

Proposition

A locale is stably compact if and only if it is a retract of a spectral locale.

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Corollary

The category of stably compact locales is equivalent to the splitting of idempotents $\mathbf{Split}(\mathbf{Spec})$ *of the category* $\mathbf{Spec}$ *of spectral locales.*

- An object of $\mathbf{Split}(\mathbf{Spec})$ is an idempotent (i.e. $f\colon X \to X$ s.t. $f \circ f = f$) in $\mathbf{Spec}$.
- A morphism $g\colon (f\colon X \to X) \to (f'\colon X' \to X')$ in $\mathbf{Split}(\mathbf{Spec})$ is a continuous map $g\colon X \to X'$ in $\mathbf{Spec}$ such that $f' \circ g = g = g \circ f$.

Spectral locales

A relation $r \subseteq D \times D'$ between distributive lattices D and D' is **approximable** if

1. $ra \stackrel{\text{def}}{=} \{b \in D' \mid a\ r\ b\}$ is a filter,
2. $r^{-}b \stackrel{\text{def}}{=} \{a \in D \mid a\ r\ b\}$ is an ideal,
3. $a\ r\ 0' \implies a = 0$,
4. $a\ r\ b \vee' c \implies (\exists b', c' \in D)\, a \leq b' \vee c' \ \&\ b'\ r\ b \ \&\ c'\ r\ c$.

Distributive lattices and approximable relations form a category $\mathbf{DL_{AP}}$.

Proposition

The category $\mathbf{DL_{AP}}$ is equivalent to the category of spectral locales.

Strong proximity lattices (Jung & Sünderhauf 1996)

A **strong proximity lattice** is an object of $\mathbf{Split}(\mathbf{DL_{AP}})$, i.e. a distributive lattice D equipped with an idempotent relation $\prec\, \subseteq S \times S$ such that

1. $\downarrow a \overset{\text{def}}{=} \{b \in D \mid b \prec a\}$ is an ideal,
2. $a \prec 0 \implies a = 0$,
3. $a \prec b \vee c \implies (\exists b' \prec b)\,(\exists c' \prec c)\, a \leq b' \vee c'$,
4. $\uparrow a \overset{\text{def}}{=} \{b \in D \mid b \succ a\}$ is a filter,

Remark The stably compact locale represented by a proximity lattice $(D, \prec)$ is the collection of **rounded ideals** of $(D, \prec)$, where an ideal $I \subseteq D$ is **rounded** if

$$a \in I \iff (\exists b \succ a)\, b \in I.$$

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4. $\uparrow a \stackrel{\text{def}}{=} \{b \in D \mid b \succ a\}$ is a filter,
5. $1 \prec a \implies a = 1$,
6. $a \wedge b \prec c \implies (\exists a' \succ a)\,(\exists b' \succ b)\, a' \wedge b' \leq c$.

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Continuous entailment relations (cf. Coquand & Zhang 2003)

An **entailment relation** on a set S is a binary relation $\vdash$ on the finite subsets of S such that

$$\frac{a \in S}{a \vdash a} \qquad \frac{A \vdash B}{A', A \vdash B, B'} \qquad \frac{A \vdash B, a \quad a, A \vdash B}{A \vdash B}$$

where a denotes $\{a\}$ and "A, B" denotes $A \cup B$.

Remark Every entailment relation $(S, \vdash)$ presents a distributive lattice with generators S and relations $\bigwedge A \leq \bigvee B$ for each $A \vdash B$.

An entailment relation $(S, \vdash)$ is **continuous** if it is equipped with an idempotent relation $\prec$ on S such that

$$(\exists C \in \mathrm{Fin}(S))\, A \prec_U C \vdash B \iff (\exists D \in \mathrm{Fin}(S))\, A \vdash D \prec_L B$$

where

$$A \prec_U B \overset{\text{def}}{\iff} (\forall b \in B)\,(\exists a \in A)\, a \prec b$$

$$A \prec_L B \overset{\text{def}}{\iff} (\forall a \in A)\,(\exists b \in B)\, a \prec b.$$

Continuous entailment relations

Proposition

The category of continuous entailment relations is equivalent to the category of strong proximity lattices.

Proof.

- If $(D, \prec)$ is a strong proximity lattice, then $(D, \vdash_D)$ defined by

$$A \vdash_D B \overset{\text{def}}{\iff} \bigwedge A \leq_D \bigvee B$$

  together with $\prec$ is a continuous entailment relation.
- If $(S, \vdash, \prec)$ is a continuous entailment relation, then the lattice D_S generated by $(S, \vdash)$ together with the relation $\ll$ on D_S defined by

$$\bigvee_{i<N} \bigwedge A_i \ll \bigwedge_{j<M} \bigvee B_j \overset{\text{def}}{\iff} \forall i < N \forall j < M \exists C \, [A_i \prec_U C \vdash B_j]$$

  is a strong proximity lattice. □

Presentation by axioms

Let R be a set of pairs of finite subsets of a set S (R: set of **axioms**).

An entailment relation $(S, \vdash)$ is **generated** by R if it is the smallest entailment relation on S that contains R, i.e. $\vdash$ is generated by

$$\frac{(A,B) \in R}{A \vdash B} \qquad \frac{a \in S}{a \vdash a} \qquad \frac{A \vdash B}{A', A \vdash B, B'} \qquad \frac{A \vdash B, a \quad a, A \vdash B}{A \vdash B}$$

Proposition

Let $(S, \vdash)$ be the entailment relation generated by a set R of axioms, and let $\prec$ be an idempotent relation on S. Then $(S, \vdash, \prec)$ is continuous if and only if

1. $A \prec_U C \,\&\, (C,D) \in R \implies (\exists E \in \mathrm{Fin}(S))\, A \vdash E \prec_L D$
2. $(C,D) \in R \,\&\, D \prec_L B \implies (\exists E \in \mathrm{Fin}(S))\, C \prec_U E \vdash B$

Presentation by axioms

A **model** of a continuous entailment relation $(S, \vdash, \prec)$ is a subset $\alpha \subseteq S$ such that

1. $A \vdash B \ \& \ A \subseteq \alpha \implies (\exists b \in B)\, b \in \alpha$;
2. $a \in \alpha \iff (\exists b \prec a)\, b \in \alpha$.

Example

If X is a locale presented by a strong proximity lattice $(D, \prec)$, the **Scott topology** $\Sigma(X)$ can be presented by a continuous entailment relation

$$(D, \vdash_\Sigma, \succ),$$

where $\vdash_\Sigma$ is generated by the axioms:

$$\vdash_\Sigma 0 \qquad\qquad a, b \vdash_\Sigma a \vee b \qquad\qquad a \vdash_\Sigma b \quad (a \geq b)$$

The models of $(D, \vdash_\Sigma, \succ)$ are rounded ideals of $(D, \prec)$.

Intrinsic duality

- Strong proximity lattice $(D, 0, \vee, 1, \wedge, \prec)$:
  1. $\downarrow a = \{b \in D \mid b \prec a\}$ is an ideal,
  2. $a \prec 0 \implies a = 0$,
  3. $a \prec b \vee c \implies (\exists b' \prec b)\,(\exists c' \prec c)\, a \leq b' \vee c'$,

  4–6. The dual properties for 1 and $\wedge$.

  The **dual** $(D, 1, \wedge, 0, \vee, \succ)$ of $(D, \prec)$ is a strong proximity lattice.

- Continuous entailment relation $(S, \vdash, \prec)$:

$$\left(\frac{(A,B) \in R}{A \vdash B}\right) \qquad \frac{a \in S}{a \vdash a} \qquad \frac{A \vdash B}{A', A \vdash B, B'} \qquad \frac{A \vdash B, a \quad a, A \vdash B}{A \vdash B}$$

$$(\exists C \in \mathrm{Fin}(S))\, A \prec_U C \vdash B \iff (\exists D \in \mathrm{Fin}(S))\, A \vdash D \prec_L B$$

The **dual** $(S, \dashv, \succ)$ of $(S, \vdash, \prec)$ is a continuous entailment relation.

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Proposition

The equivalence between continuous entailment relations and strong proximity lattices commutes with the dualities.

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Question

If X is the stably compact locale presented by $(D, \prec)$, does $(D^{\mathsf{d}}, \succ)$ present the de Groot dual of X?

De Groot duality

Definition (Escardó 2000)

The de Groot dual X^{d} of a stably compact locale X is the collection of Scott open filters on X.

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Note. Scott open filters on $X \cong$ models of $\mathrm{P_U}(X)$ where $\mathrm{P_U}(X)$ is the **upper powerlocale** of X.

Lemma

If X and Y are stably compact locales, then

$$X^{\mathsf{d}} \cong Y \iff \textit{models of } \mathrm{P_U}(X) \cong \textit{models of } \Sigma(Y).$$

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Lemma

If X and Y are stably compact locales, then

$$X^{\mathsf{d}} \cong Y \iff \mathrm{P_U}(X) \cong \Sigma(Y).$$

Theorem

Let $(D, \prec)$ be a strong proximity lattice, and X and Y be stably compact locales presented by $(D, \prec)$ and $(D^{\mathsf{d}}, \succ)$ respectively. Then

$$\mathrm{P_U}(X) \cong \Sigma(Y).$$

De Groot duality in strong proximity lattices

Theorem
Let X and Y be stably compact locales presented by strong proximity lattices $(D, \prec)$ and $(D^{\mathsf{d}}, \succ)$ respectively. Then $\mathrm{P_U}(X) \cong \Sigma(Y)$.

Proof.

- The upper powerlocale $\mathrm{P_U}(X)$ is presented by an entailment relation on D generated by

$$\vdash 1 \qquad a, b \vdash a \wedge b \qquad a \vdash b \quad (a \leq b)$$

  with an idempotent relation $\prec$.

- The Scott topology $\Sigma(X)$ is presented by an entailment relation on D generated by

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  with an idempotent relation $\succ$.

De Groot duality in strong proximity lattices

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  with an idempotent relation $\prec$. □

Vietoris powerlocales $\mathrm{P_V}(X)$

Proposition

If X is stably compact then $\mathrm{P_V}(X)^{\mathsf{d}} \cong \mathrm{P_V}(X^{\mathsf{d}})$.

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Proposition

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Proof.

Let X be a stably compact locale presented by a strong proximity lattice $(D, \prec)$.

The Vietoris powerlocale $\mathrm{P_V}(X)$ is presented by an entailment relation on $\{\Diamond a \mid a \in D\} \cup \{\Box a \mid a \in D\}$ generated by

$$\Diamond 0 \vdash \qquad \Diamond(a \vee b) \vdash \Diamond a, \Diamond b \qquad \Diamond a \vdash \Diamond b \quad (a \leq b)$$

$$\vdash \Box 1 \qquad \Box a, \Box b \vdash \Box(a \wedge b) \qquad \Box a \vdash \Box b \quad (a \leq b)$$

$$\Box a, \Diamond b \vdash \Diamond(a \wedge b)$$

$$\Box(a \vee b) \vdash \Box a, \Diamond b$$

The idempotent relation associated with $\mathrm{P_V}(X)$ is

$$\Diamond a \prec \Diamond b \overset{\text{def}}{\iff} a \prec b, \qquad \Box a \prec \Box b \overset{\text{def}}{\iff} a \prec b.$$

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$$\Diamond a, \Box b \vdash \Diamond(a \vee b)$$

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$$\Diamond a \prec \Diamond b \overset{\text{def}}{\iff} a \succ b, \qquad \Box a \prec \Box b \overset{\text{def}}{\iff} a \succ b. \qquad \Box$$

The space of valuations

Definition
A **probabilistic valuation** on a locale X is a Scott continuous map $\mu\colon \Omega(X) \to [0,1]_{\mathcal{L}}$ to the lower reals $[0,1]_{\mathcal{L}}$ satisfying $\mu(0) = 0$, $\mu(1) = 1$, and the modular law: $\mu(x) + \mu(y) = \mu(x \wedge y) + \mu(x \vee y)$.

A **covaluation** on X is a Scott continuous map $\nu\colon \Omega(X) \to [0,1]_{\mathcal{U}}$ to the upper reals $[0,1]_{\mathcal{U}}$ satisfying $\nu(1) = 0$, $\nu(0) = 1$, and the modular law.

- The **space of valuations** $\mathfrak{V}(X)$ is a locale whose models are valuations on X.
- The **space of covaluations** $\mathfrak{C}(X)$ is a locale whose models are covaluations on X.

The space of valuations

If X is a locale presented by a strong proximity lattice $(D, \prec)$, then the space of valuations $\mathfrak{V}(X)$ is presented by an entailment relation on

$$S = \{\langle p, a \rangle \mid p \in \mathbb{Q}, a \in D\}$$

generated by the axioms

$$\begin{array}{llll}
\emptyset \vdash \langle p, 0 \rangle & (p < 0) & \langle p, 0 \rangle \vdash \emptyset & (0 < p) \\
\emptyset \vdash \langle p, 1 \rangle & (p < 1) & \langle p, 1 \rangle \vdash \emptyset & (1 < p) \\
\langle p, a \rangle \vdash \langle q, b \rangle & & & (p \geq q \ \& \ a \leq b) \\
\langle p, a \rangle, \langle q, b \rangle \dashv\vdash \langle r, a \wedge b \rangle, \langle s, a \vee b \rangle & & & (p + q = r + s)
\end{array}$$

with an idempotent relation $\langle p, a \rangle \prec_{\mathfrak{V}} \langle q, b \rangle \overset{\text{def}}{\iff} p > q \ \& \ a \prec b$.

Note. A model $m \subseteq S$ of $(S, \vdash, \prec_{\mathfrak{V}})$ corresponds to a valuation μ on the ideals of $(D, \prec)$ given by

$$\mu(I) = \sup_{a \in I} \sup_{\langle p, a \rangle \in m} p.$$

so that $\langle p, a \rangle \in m \iff p < \mu(a)$.

Proposition

If X is a stably compact locale, then $\mathfrak{V}(X)^{\mathsf{d}} \cong \mathfrak{C}(X^{\mathsf{d}})$.

Proof.

Suppose X is presented by a strong proximity lattice $(D, \prec)$. Then $\mathfrak{V}(X)$ is presented by an entailment relation on

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and an idempotent relation $\langle p, a\rangle \prec_{\mathfrak{V}^{\mathsf{d}}} \langle q, b\rangle \stackrel{\text{def}}{\iff} p < q \ \& \ a \succ b$.

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