# Glivenko's theorem, finite height, and local finiteness

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|---|---|---|---|
| (Glivenko, 1929) | $\mathrm{CL} \vdash \varphi$ | iff | $\mathrm{IL} \vdash \neg\neg\varphi$ |
| (Matsumoto, 1955) | $\mathrm{S5} \vdash \varphi$ | iff | $\mathrm{S4} \vdash \neg\Box\neg\Box\varphi$ |

**In Kripke semantics:**

$\mathrm{IL}$ is the logic of partial orders,
$\mathrm{CL}$ is the logic of singletons, which are partial orders of height 1.

$\mathrm{S4}$ is the logic of preorders,
$\mathrm{S5}$ is the logic of equivalence relations, which are preorders of height 1.

Let $\mathrm{L}[h]$ be the extension of a logic $\mathrm{L}$ with the axiom of height $h$.

$$\mathrm{IL}[1] \vdash \varphi \text{ iff } \mathrm{IL} \vdash \neg\neg\varphi, \quad \mathrm{S4}[1] \vdash \varphi \text{ iff } \mathrm{S4} \vdash \Diamond\Box\varphi.$$

**This talk:**

- Generalization for arbitrary finite height
- and for non-transitive and polymodal logics.
- Interplay with local finiteness.

## Formulas and logics of finite height

intermediate: $b^{\mathrm{I}}_0 = \bot, \qquad b^{\mathrm{I}}_{i+1} = p_{i+1} \vee (p_{i+1} \to b^{\mathrm{I}}_i)$

modal: $b_0 = \bot, \qquad b_{i+1} = p_{i+1} \to \Box(\Diamond p_{i+1} \vee b_i)$

$\mathrm{L}[h]$ extends $\mathrm{L}$ with the formula of height $h$. In particular,

$$\mathrm{IL}[1] = \mathrm{CL}, \quad \mathrm{S4}[1] = \mathrm{S5}$$

$$\mathrm{IL}[1] \vdash \varphi \text{ iff } \mathrm{IL} \vdash \neg\neg\varphi \qquad \mathrm{S4}[1] \vdash \varphi \text{ iff } \mathrm{S4} \vdash \Diamond\Box\varphi$$

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$\mathrm{IL}[2] \vdash \varphi$ iff $\mathrm{IL} \vdash$ ? $\qquad$ $\mathrm{S4}[2] \vdash \varphi$ iff $\mathrm{S4} \vdash$ ?

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$\dots$ $\qquad$ $\dots$

## First result

Such translations exist for all $h$ in the finite-variable case:

A *k-formula* is a formula in variables $p_0, \dots p_{k-1}$.
For all $h, k < \omega$ there exists a translation (a formula with a parameter) $\mathrm{tr}(\cdot)$, and its intermediate analogue $\mathrm{tr}^{\mathrm{I}}(\cdot)$, such that for all $k$-formulas $\varphi$

$$\mathrm{IL}[h] \vdash \varphi \text{ iff } \mathrm{IL} \vdash \mathrm{tr}^{\mathrm{I}}(\varphi) \qquad \mathrm{S4}[h] \vdash \varphi \text{ iff } \mathrm{S4} \vdash \mathrm{tr}(\varphi)$$

The *k-canonical frame* of a logic $\mathrm{L}$ (the representation of the $k$-generated free algebra of $\mathrm{L}$) is built from maximal $\mathrm{L}$-consistent sets of $k$-formulas.

**Theorem (Shehtman, 1978)**

Let $k < \omega$. There exist formulas $\mathbf{B}_{h,k}$ (and their intuitionistic analogs $\mathbf{B}^{\mathrm{I}}_{h,k}$) such that for every $x$ in the $k$-canonical frame $\mathrm{F}_k$ of $\mathrm{S4}$ (of $\mathrm{Int}$)

$$\mathbf{B}_{h,k} \in x \quad \Leftrightarrow \quad \text{the depth of } x \text{ in } \mathrm{F}_k \text{ is less than or equal to } h.$$

Moreover, if $x$ is of infinite depth, then $\Diamond \mathbf{B}_{h,k} \in x$ for all $h < \omega$; that is to say, $\mathrm{F}$ is *top-heavy*.

The term ‘top-heavy’ is due to Fine (1985).

**Theorem (First result)**

Let $k < \omega$. For all $k$-formulas $\varphi$ we have:

- $\mathrm{IL}[h+1] \vdash \varphi \ \Leftrightarrow\ \mathrm{IL} \vdash \bigwedge_{i \leq h}((\varphi \to \mathbf{B}^{\mathrm{I}}_{i,k}) \to \mathbf{B}^{\mathrm{I}}_{i,k})$;
- $\mathrm{S4}[h+1] \vdash \varphi \ \Leftrightarrow\ \mathrm{S4} \vdash \bigwedge_{i \leq h}(\Box(\Box\varphi \to \mathbf{B}_{i,k}) \to \mathbf{B}_{i,k})$.

In particular, for $h = 0$ the formulas $\mathbf{B}_{0,k}$ and $\mathbf{B}^{\mathrm{I}}_{0,k}$ are $\bot$ for all $k < \omega$:

$$\mathrm{IL}[1] \vdash \varphi \text{ iff } \mathrm{IL} \vdash \neg\neg\varphi, \quad \mathrm{S4}[1] \vdash \varphi \text{ iff } \mathrm{S4} \vdash \Diamond\Box\varphi.$$

The first result is based on top-heaviness of canonical frames.

In turn, top-heaviness can be obtained for the case when finite-height extensions are locally tabular (poly)modal logics.

A logic is said to be *$k$-tabular* if, up to the equivalence in it, there exist only finitely many $k$-formulas. A logic is *locally tabular* (or *locally finite*) if it is $k$-tabular for every finite $k$.

Segerberg, 1971; Maksimova, 1975: A transitive logic is locally tabular iff it is of finite height. In particular, all $\mathrm{S4}[h]$ are locally tabular.

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Shetman, Sh, 2016: Every 1-tabular (a fortiori, locally tabular) modal logic is a pretransitive logic of finite height.

$\mathrm{L}$ is *pretransitive* if there is a formula $\Diamond^*(p)$ ('master modality') s.t. $\Diamond^*(\varphi)$ expresses the satisfiability of $\varphi$ in cones on models of $\mathrm{L}$.

## Second result

Let $\mathrm{L}$ be a pretransitive logic, $h, k < \omega$. If $\mathrm{L}[h]$ is $k$-tabular, then exists a translation $\mathrm{tr}(\cdot)$ s.t. for all $k$-formulas $\varphi$

$$\mathrm{L}[h+1] \vdash \varphi \text{ iff } \mathrm{L} \vdash \mathrm{tr}(\varphi)$$

All 1-tabular (a fortiori, locally tabular) logics are pretransitive.

$\mathrm{L}$ is *pretransitive* if there is a formula $\Diamond^*(p)$ ('master modality') s.t. $\Diamond^*(\varphi)$ expresses the satisfiability of $\varphi$ in cones on models of $\mathrm{L}$.
Synonyms: EDPC-logics (Blok and Pigozzi), logics with expressible master modality (Kracht), conically expressive logics (Shehtman).

Kowalski and Kracht, 2006: $\mathrm{L}$ is pretransitive iff $\mathrm{L}$ is *m-transitive* for some $m \geq 0$, i.e., contains $\Diamond^{m+1}p \to p \vee \Diamond p \vee \ldots \vee \Diamond^m p$.

$m$-transitivity says "if $y$ is accessible from $x$ in $m+1$ steps, then $y$ is accessible from $x$ in $\leq m$ steps"

$\Diamond^*(\varphi)$ is $\varphi \vee \Diamond\varphi \vee \ldots \vee \Diamond^m\varphi$

In the polymodal case, $\Diamond p$ is $\bigvee \Diamond_i p$

## Examples of pretransitive logics

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| $\mathrm{K4},\ \mathrm{wK4} = [\Diamond\Diamond p \to \Diamond p \vee p]$ | 1-transitive |
| $\mathrm{K5} = [\Diamond p \to \Box\Diamond p]$ | 2-transitive |
| $[\Diamond^n p \to \Diamond^m p]$, $n > m$ | $(n-1)$-transitive |
| The (expanding) product of two transitive logics | 2-transitive |

The *height of a polymodal frame* $(W, (R_i)_{i<n})$ is the height of the preorder $(W, (\bigcup_{i<n} R_i)^*)$.
In the pretransitive case, the formulas of finite height can be defined:
$B_0 = \bot,\ B_h = p_h \to \Box^*(\Diamond^* p_h \vee B_{h-1})$.

**Theorem (Second result)**

*Let* $\mathrm{L}$ *be a pretransitive logic,* $h, k < \omega$*. If* $\mathrm{L}[h]$ *is* $k$*-tabular, then:*

(a) *For every* $i \leq h$*, there exists a formula* $\mathbf{B}_{i,k}$ *such that* $\mathbf{B}_{i,k} \in x$ *iff the depth of* $x$ *in the* $k$*-canonical frame of* $\mathrm{L}$ *is less than or equal to* $i$*.*

(b) *For all* $k$*-formulas* $\varphi$*,*

$$\mathrm{L}[h+1] \vdash \varphi \;\Leftrightarrow\; \mathrm{L} \vdash \bigwedge_{i \leq h} (\Box^*(\Box^*\varphi \to \mathbf{B}_{i,k}) \to \mathbf{B}_{i,k}).$$

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In particular, the inconsistent logic $\mathrm{L}[0]$ is locally tabular, hence for every pretransitive $\mathrm{L}$

$$\mathrm{L}[1] \vdash \varphi \text{ iff } \mathrm{L} \vdash \Diamond^*\Box^*\varphi \tag{1}$$

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Kudinov, Sh, 2011: A more direct (syntactic) proof of (1).

There is a simple semantic explanation of (1) based on "maximality property" of pretransitive canonical frames...

$$\mathrm{IL}[1] \vdash \varphi \text{ iff } \mathrm{IL} \vdash \neg\neg\varphi, \quad \mathrm{S4}[1] \vdash \varphi \text{ iff } \mathrm{S4} \vdash \Diamond\Box\varphi.$$

Proof. Immediate form the finite model property:
Every point in a finite poset sees a model of $\mathrm{CL}$, a maximal point.

Likewise for $\mathrm{S4}$ and $\mathrm{S5}$.

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In general, no FMP...

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In general, no FMP... Instead, we have:

**Lemma (Esakia-Fine lemma for pretransitive canonical frames)**

*If $\varphi \in x \in W$, then $R^*(x) \cap \|\varphi\|$ has a maximal element.*

Proof of (1). Immediate form the above maximality property: put $\varphi = \top$.
Every point in the $\omega$-canonical frame sees a model of $\mathrm{L}[1]$, a maximal $R^*$-cluster.

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$$\mathrm{L}[h+1] \vdash \varphi \text{ iff } \mathrm{L} \vdash \mathrm{tr}(\varphi)$$

We can construct $\mathrm{tr}(\cdot)$ for $k$-formulas whenever every point in the $k$-canonical frame sees a model of $\mathrm{L}[h+1]$. This is true whenever $\mathrm{L}[h]$ is $k$-tabular...

Let $\mathrm{F} = (W, (R_i)_{i<n})$ be the $k$-canonical frame of a pretransitive logic $\mathrm{L}$. Let $R = \bigcup_{i<n} R_i$, and let $W[\leq h]$ be the set of points of depth $\leq h$.

$\mathrm{F}$ is *h-heavy* if every its point of depth $> h$ is $R^*$-related to a point of depth $h$.

**Theorem**

*If $\mathrm{L}[h]$ is $k$-tabular, then:*

1. *Each element of $W[\leq h]$ is definable in $\mathrm{F}$.*
2. *For $i \leq h$, the set $W[\leq i]$ is definable in $\mathrm{F}$.*
3. *$\mathrm{F}$ is $(h+1)$-heavy.*

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3. *$\mathrm{F}$ is $(h+1)$-heavy.*

Let $\mathbf{B}_{i,k}$ define $W[{\leq}i]$. Then it is (almost) straightforward that

$$\mathrm{L}[h+1] \vdash \varphi \;\Leftrightarrow\; \mathrm{L} \vdash \bigwedge_{i \leq h} (\Box^*(\Box^*\varphi \to \mathbf{B}_{i,k}) \to \mathbf{B}_{i,k}).$$

local tabularity of finite-height extensions $\Rightarrow$

top-heaviness of finitely generated canonical frames $\Rightarrow$

translations for arbitrary finite height local tabularity of finite-height extensions $\Rightarrow$

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translations for arbitrary finite height

$k$-tabularity of $\mathrm{L}[h]$ $\Rightarrow$

$(h+1)$-heaviness of the $k$-generated canonical frame $\Rightarrow$

translation for $k$-fragment of $L[h+1]$

Segerberg, 1971; Maksimova, 1975:
A transitive logic is locally tabular iff it is of finite height iff it is 1-tabular.

Shehtman, Sh, 2016:
Every 1-tabular logic is a pretransitive logic of finite height.

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Every 1-tabular logic is a pretransitive logic of finite height.

In general, the converse is not true!
Makinson, 1981:
There exists a pretransitive $\mathrm{L}$ s.t. none of the logics $\mathrm{L}[h]$, $h > 0$, are 1-tabular:
put $\mathrm{L} = [\Diamond^3 p \to \Diamond^2 p]$.

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## In general, $k$-tabularity of $\mathrm{L}[h]$ depends on $h$ and $k$.

**Example**

Let $\mathrm{L} = [p \to \Diamond p,\ \Diamond^3 p \to \Diamond^2 p,\ \Box^2\Diamond^2 p \to \Diamond^2\Box^2 p]$.
Then $\mathrm{L}[1] \vdash p \leftrightarrow \Box p$. Thus $\mathrm{L}[1]$ is locally tabular (and we have translations from $\mathrm{L}[2]$ to $\mathrm{L}$ for all $k < \omega$). But $\mathrm{L}[2]$ is not 1-tabular.

## $k$-tabularity $\Rightarrow$ local tabularity?

Maksimova, 1975:
A unimodal transitive logic is locally tabular iff it is 1-tabular.

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This equivalence holds for many others families of modal logics.

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Maksimova, 1975:
A unimodal transitive logic is locally tabular iff it is 1-tabular.

Shehtman, Sh, 2016:
This equivalence holds for many others families of modal logics.

But it does not hold in general...

**Theorem**

*There exists a unimodal 1-tabular logic which is not locally tabular.*

**Proof.**

Let $\mathrm{F} = (\omega + 1, R)$, where $xRy$ iff $x \leq y$ or $x = \omega$.
1-tabularity of the logic of $\mathrm{F}$ is a straightforward exercise.

**Lemma (Shehtman, Sh, 2016)** If the logic of a frame is locally tabular, then the logic of any its subframe is locally tabular.

The restriction of the cluster $(\omega + 1, R)$ onto $\omega$ is the frame $(\omega, \leq)$, which is of infinite height. Thus $Log\,(\omega + 1, R)$ is not locally tabular. $\square$

1-tabularity does not imply local tabularity.

### Question

It is unknown whether 2-tabularity of a modal logic implies its local tabularity.
At least, does $k$-tabularity imply local tabularity, for some fixed $k$ for all modal logics?

The same questions are open in the intuitionistic case.

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*Thank you!*

Glivenko's theorem in superintuitionistic, modal, and intuitionistic modal logics:


- V. Glivenko, *Sur quelques points de la logique de M. Brouwer*, 1929
- K. Matsumoto, *Reduction theorem in Lewis's sentential calculi*, 1955
- V. Rybakov, *A modal analog for Glivenko's theorem and its applications*, 1992
- G. Bezhanishvili, *Glivenko type theorems for intuitionistic modal logics*, 2001
- A. Kudinov and Sh, *Finite model property of pretransitive analogs of* $\mathrm{S5}$, 2011
- T. Litak, M. Polzer, and U. Rabenstein, *Negative Translations and Normal Modality* , 2017
- ...
- This talk:
  Sh, *Glivenko's theorem, finite height, and local finiteness*
  arxiv.org/abs/1806.06899