Stretching of Polymers

Two dimensional turbulence with polymer additives

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2d turbulence

\[
\partial_t \omega + u \cdot \nabla \omega = \nu \nabla^2 \omega - \mu \omega + F_\omega
\]

\[
E(k) \sim k^{-5/3}
\]

\[
E(k) \sim k^{-3 + \delta(\mu)}
\]

How polymer additives affect forward and inverse cascade?

Perlekar et al., PRL (2011); Ray et al., PRL (2011); Boffetta et al., ARFM (2012).

Wiki: Linear polymer molecule
2d turbulence: Topological structures

\[ \Lambda = \frac{(\omega^2 - \sigma^2)}{4} \]

Expts: Daniel and Rutgers, PRL (2002);

How polymer additives affect the topological properties?
FIG. 1. Experimental setup. A voltage difference $V = V^+ - V^-$ is applied to the film generating a uniform current density $J$. Beneath the film is a set of bar magnets with alternating poles.

Kolmogorov forcing generates turbulence in soap-films.

Jun et al., PRL, 96, 024502 (2006)
For comparison, an independent run was also carried out for 15 ppm. Figure 5(b) shows that, in the absence of polymer, the situation is somewhat different; that the time criterion is strictly obeyed in the experiment. A significant elimination of strong saddles and the weakening of saddles by polymer stretching has the drastic effect of quenching turbulence, it suggests that this hydrodynamic structure may play a role in the onset of the turbulent suppression and the higher energy transfer rate. It forms a plateau for a small range of injection voltages. This measurement suggests that there exist two thresholds between 50 and 65 volts. This measurement shows that when \( \phi \) is fixed, two thresholds can be identified, \( \varepsilon_p \) and \( \varepsilon_\alpha \) corresponding to the saturation of the elastic deformation and the onset of the turbulent suppression and the higher energy transfer rate, respectively.

Since polymer-turbulence interactions are primarily via molecular viscosity, we believed that the lower threshold is determined precisely by the relation 

\[
C_1 \varepsilon \sim \varepsilon_p C_2
\]

for \( \varepsilon_p \) and \( \varepsilon_\alpha \) determined precisely by the relations 

\[
C_1 \varepsilon \sim \varepsilon_\alpha C_2
\]

and 

\[
C_1 \varepsilon \sim \varepsilon_v C_2
\]

where \( C_1 \) and \( C_2 \) are constants. It is interesting to note that when \( \varepsilon \) is increased, the fraction of energy consumed by the fluid viscosity remains constant, it follows that 

\[
\varepsilon_v \sim \varepsilon_{\text{rms}}
\]

and 

\[
\varepsilon_{\text{rms}} \sim \varepsilon_{\text{rms}}
\]

is a smooth increasing function of \( \phi / \varepsilon \). The effect is more dramatic for the energy transfer rate and 

\[
\varepsilon \sim \varepsilon (0.0030
\]

(see Fig. 2) 

\[
85
\]

(10 ppm) until 

\[
28
\]

(2006). Figure 5(b) shows that, in the absence of polymer, the situations are somewhat different, but the transitions in this case are much weaker. We found that for a given injection voltage, the polymer effects on forced 2D turbulence are more pronounced than in 3D as well as in 2D turbulence. The above measurement shows that for a given injection voltage, the polymer effects on forced 2D turbulence are more pronounced than in 3D as well as in 2D turbulence. The above measurement shows that for a given injection voltage, the polymer effects on forced 2D turbulence are more pronounced than in 3D as well as in 2D turbulence. The above measurement shows that for a given injection voltage, the polymer effects on forced 2D turbulence are more pronounced than in 3D as well as in 2D turbulence.
Suppression of both large and small scales

\[ S_2(l) \text{ (cm}^2/\text{s}^2) \]

\[ l \text{ (cm)} \]

100
10
1

1.8

\[ l_{\text{inj}} \]
Soap-film experiment-4/4

The EM cell is well suited for this task since the full velocity needs to be calculated. To start with, we used the PRL(2006)024502. We measured the overall energy budget of the system. The rate of energy uptake and the energy dissipation rates due to fluid viscosity match if the polymer-fluid interaction is not included in the equation. The significance of this observation is demonstrated if we compare the point where the two groups of pdfs cross in Fig. 3(a) and Fig. 4(b). The air friction coefficient was kept constant, whereas the point where the two groups of pdfs cross is found that in the absence of polymer, the two sides are matched if the polymer is included and will be found by Eq. (2). Such discrepancy is expected because the definitions and can be evaluated. The rate of energy uptake by the polymer was kept constant, yielding Eq. (2). Such discrepancy is expected because the definitions and can be evaluated. The rate of energy uptake by the polymer was kept constant, yielding Eq. (2). Such discrepancy is expected because the definitions and can be evaluated. The rate of energy uptake by the polymer was kept constant, yielding Eq. (2). Such discrepancy is expected because the definitions and can be evaluated. The rate of energy uptake by the polymer was kept constant, yielding Eq. (2).
Modeling polymer solutions

**FENE-P Model**

\[
\frac{\partial u_\alpha}{\partial t} + (u_\gamma \partial_\gamma) u_\alpha = -\partial_\alpha p + \nu \partial_\gamma u_\alpha + \partial_\gamma T_{\alpha\gamma}
\]

\[
\frac{\partial C_{\alpha\beta}}{\partial t} + (u_\gamma \partial_\gamma) C_{\alpha\beta} = (\partial_\gamma u_\alpha) C_{\gamma\beta} + C_{\alpha\gamma}(\partial_\gamma u_\beta) - \frac{1}{\mu} T_{\alpha\beta}
\]

\[
T_{\alpha\beta} = \mu \frac{f(r) C_{\alpha\beta} - \delta_{\alpha\beta}}{\tau_P} \quad f(r) = \frac{L^2 - 2}{L^2 - r^2}
\]

**Oldroyd-B Model**

\[
L^2 \to \infty \quad f(r) \to 1
\]

Assumption: Smooth flow around polymer.
Earlier studies: Simulations

Homogeneous isotropic turbulence, $256^3$ DNS

Oldroyd-B model

Passive polymers
$\dot{W} = \lambda \tau > 1$

Unbounded growth in polymer extension. No steady state.

Active polymers
$\dot{W} = \lambda \tau < 1$

1. Presence of back-reaction dramatically alters the steady state.
2. Steady state for polymer extension.
3. No coil-stretch transition!

Earlier studies: Simulations

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Oldroyd-B model

Passive polymers

$Wi = \lambda \tau > 1$

Unbounded growth in polymer extension. No steady state.

Active polymers

$Wi = \lambda \tau < 1$

Polymers act as large scale sink

Question raised in thesis (2003):
What happens in a well-resolved forward and inverse cascade?

$\epsilon_V = \epsilon_N - \frac{\mu}{\tau_P^2} (r^2 - 2)$

S. Mussachio, PhD Thesis
Results

Our simulations:

1. DNS of Navier-Stokes + FENE-P equations.
2. Kolmogorov forcing to generate flows similar to experiments by rescaling forcing amplitude.
3. Maintain constant energy injection rate.
PDF of velocity

\[ P(u) \]

\[ u \]

WITHOUT POLYMER
FIT: \( a \exp(-b \, u^2) \)
FIT: \( a \exp(-b \, |u|^3) \)

WITH POLYMER
FIT: \( a \exp(-b \, u^2) \)
Energy suppression
Energy spectrum: Small wave-vectors

\[ E(k) \]

\( c=0.0 \)
\( c=0.1 \)
\( c=0.4 \)

\( Wi = 1 \)

\( N=4096^2 \)
Energy spectrum: Small wave-vectors

![Graph of energy spectrum](image)

- $N=4096^2$
- $c=0.1$
- $\tau_p = 1, 2, 4$
1. Suppression of small k modes
2. Energy suppressed at intermediate wave-vectors
3. Enhancement of energy at large wave-vectors (Similar to 3D, see Perlekar et al., PRE (2010), PRL (2006))

Energy spectrum: Suppression of small k modes

\[ \log_{10} [E(k)] \]

\[ \log_{10}(k) \]

WITH POLYMER

WITHOUT POLYMER

\( k^{-2.68} \)
Beneath the film is a set of bar magnets with alternating poles.

The experiments of Y. Jun, J. Zhang, and Xiao-Lun Wu, Phys.

...set of second-order structure functions, which we determined to be Kolmogorov-like scaling large Taylor-microscale Reynolds number...
PDF of $\Lambda$

\[ \Lambda = \frac{(\omega^2 - \sigma^2)}{4} \]
Polymer extension vs vorticity
Joint PDF

\[ \Lambda \]

\[ r^2_P \]
Conclusions

1. Energy spectra is strongly modified in presence of polymers.

2. For small concentrations, the distributions of saddles and centers is not dramatically modified by polymers.

3. Regions of polymer extensions are strongly correlated with the extensional regions

4. Similarities with 3D turbulence in forward cascade