Onset and universality of turbulent drag reduction in von Karman flow

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Two effects of polymers on flow

Turbulent drag reduction-
modification of ***turbulent*** flow
at Re>>1 by adding polymers

?

Elastic turbulence-
Elastic instabilities and transition
from ***laminar to random*** flow by
adding polymers

Both effects result from interaction of polymers with flow, causing polymer stretching, and back reaction of polymers on the flow
Both flows are characterized by two fields: velocity and elastic stresses, similar to magneto-hydrodynamics, where velocity and magnetic fields are necessary to know in order to understand the flow

Two ways to investigate these phenomena:

- ***macroscopic approach to study hydrodynamics and stresses***
- ***microscopic approach to study dynamics and statistics of polymer molecules***

Three limiting cases:

$$1.\ \mathrm{Re} >> 1; Wi \Rightarrow 0$$

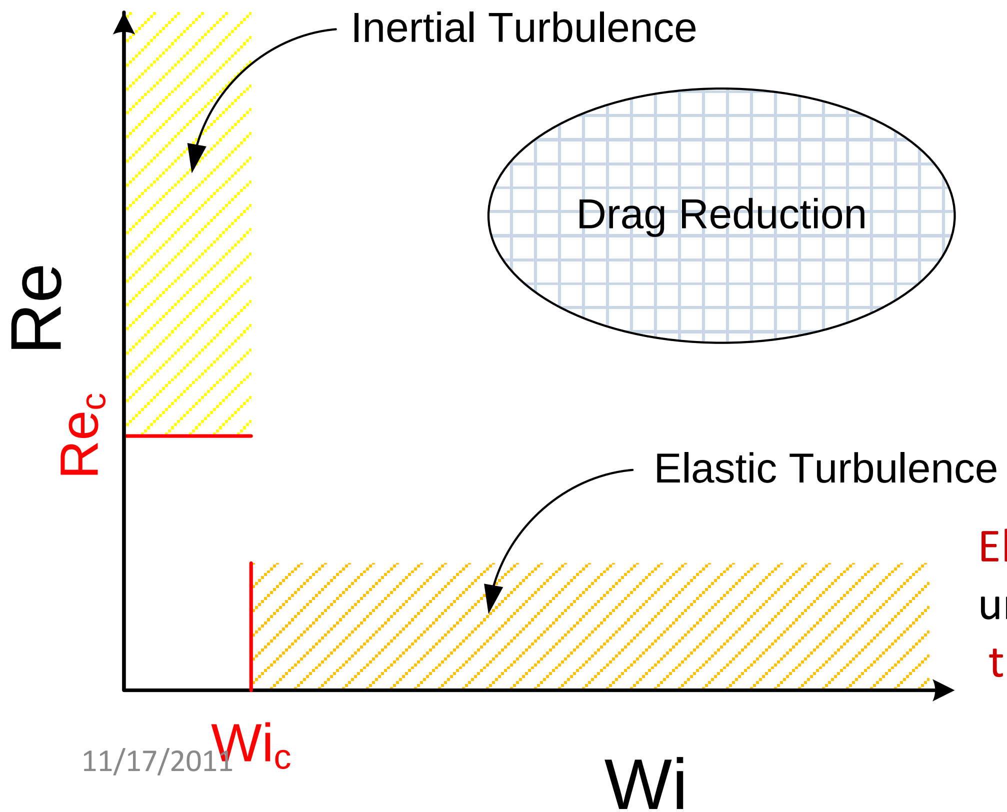
Hydrodynamic turbulence

$$2.\ Wi >> 1; \mathrm{Re} \Rightarrow 0$$

Elastic turbulence

$$3.\ \mathrm{Re} >> 1; Wi >> 1$$

Turbulent drag reduction

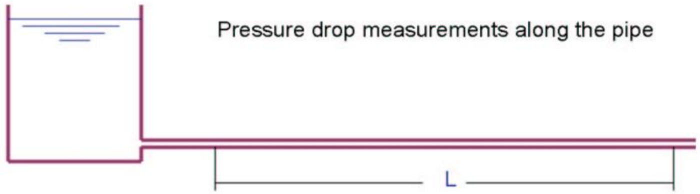


Elastic turbulence is a model system to understand more complicated phenomenon- turbulent drag reduction

Turbulent Drag Reduction by Dilute Addition of Polymers

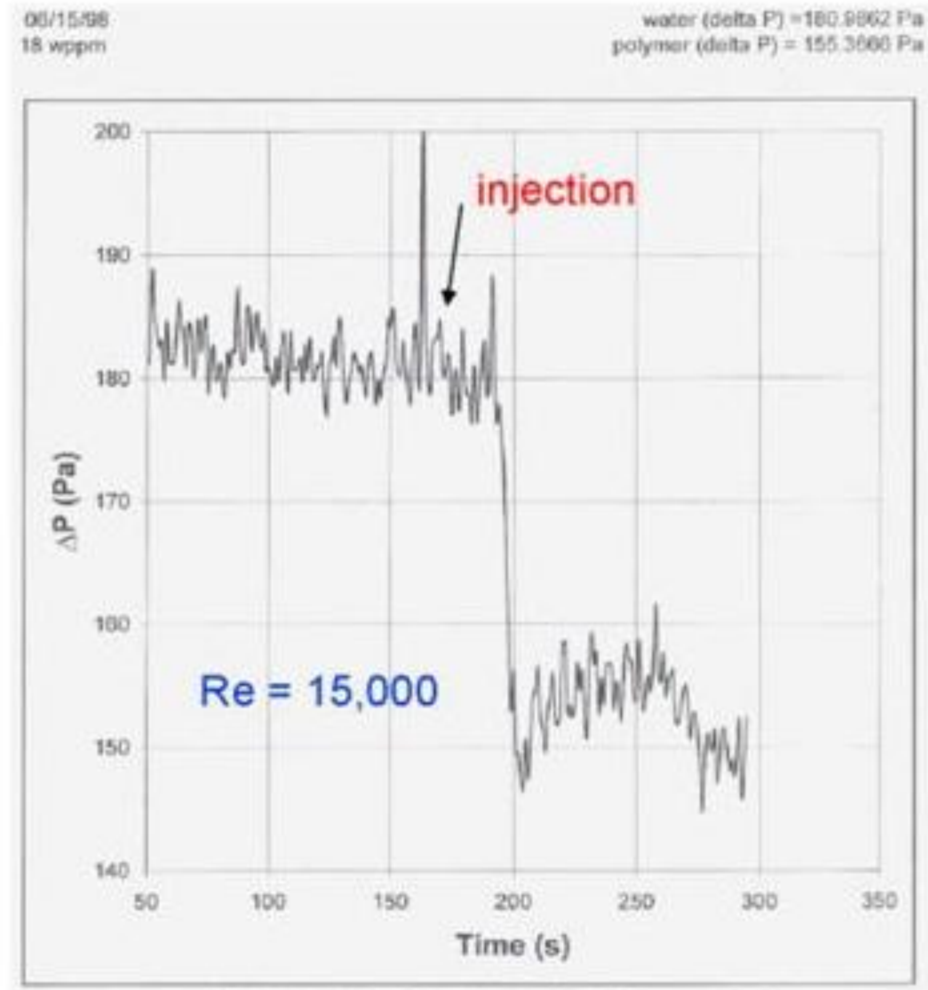
(Toms 1948)

Pressure drop measurements along the pipe



$\Delta P_s / \Delta P_w = O(1/5)$

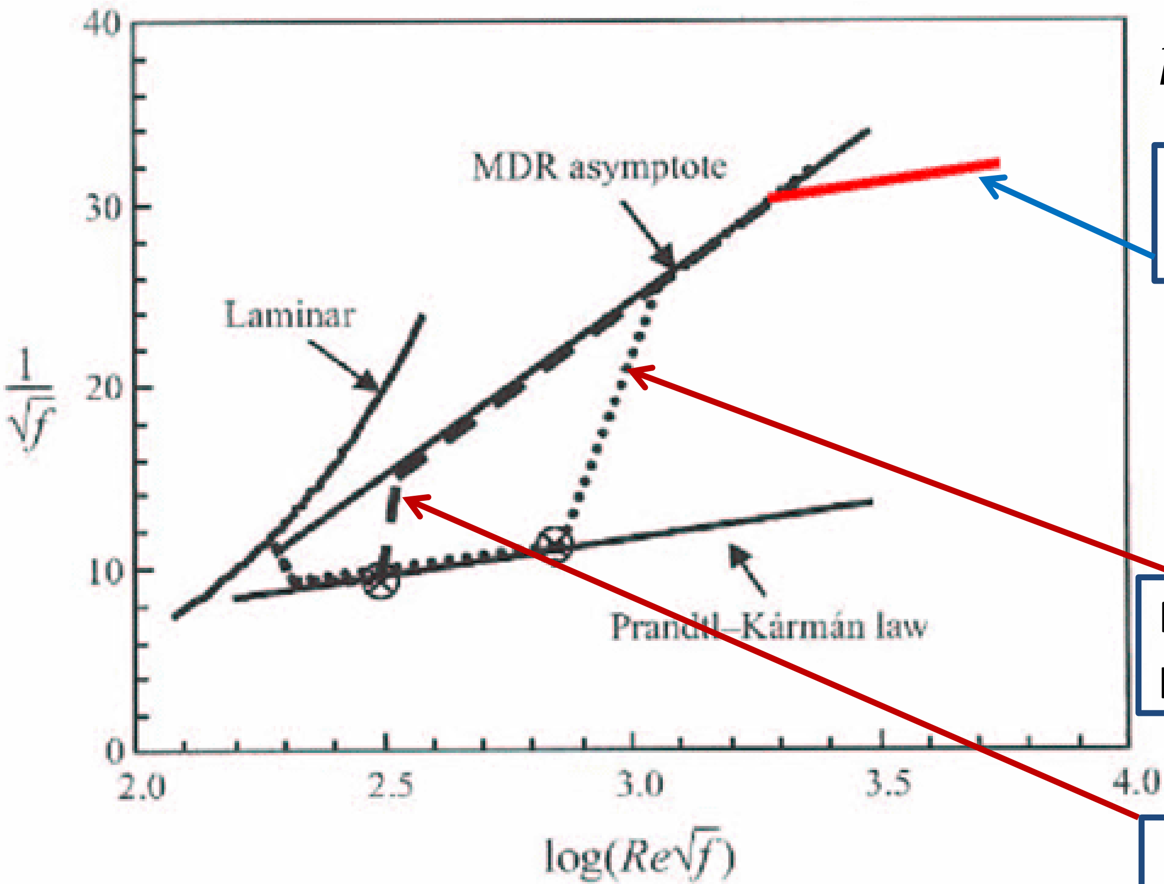
for polymer concentrations of the order of
a few tens of parts per million by weight



Pressure Drop Across ENtire Length of Pipe (148 cm)
[14% decrease in pressure drop with polymer]

Turbulent drag reduction is characterized by the friction factor that can be reduced up to 5-6 times by adding high molecular weight polymer molecules (usually M=10^7 and concentration of 10-100 ppm)

Specific aspects of the drag reduction problem



$$f = \frac{\Delta P}{\rho V^2}\frac{R}{L}; \mathrm{Re} = \frac{2VR}{\nu}$$

Saturation of drag due to full polymer stretching

- Onset
- Finite drag reduction
- MDR
- Peel-off from MDR

Friction factor for smaller polymer concentration

Friction factor for larger polymer concentration

Schematic illustration the onset of drag reduction and the maximum drag-reduction asymptote (MDR) in a pipe-empirical limit of turbulent drag reduction (Virk 1975)

Two basic approaches and their misconceptions

Lumley (~1970) and further refinements

1. ! Coil-stretch transition does occur in random flow-only then polymer can modify flow
2. ! Time criterion $\tau_p / \tau_{turb} \approx 1 \rightarrow$ Wi≈1
3. ! Since L<η-diss length, polymer "feels" just small scales→ spatially-smooth, random in time flow
4. ? Buffer layer, where polymer mostly stretched→highest extension strain rate→ highest viscosity

De Gennes (~1990) and further refinements

1. ? No coil-stretch transition in a random flow
2. ? Even partially stretched polymer stores elastic energy
3. ! Elastic energy~visc diss gives r* at which spectra is modified
4. ! Based on (3) and also balance elastic energy~turb energy gives explanation for onset of drag reduction and maximum drag reduction asymptote

Recent refinement of the Lumley's theory by I. Procaccia et al adds specific dependence of viscosity on distance from the wall to describe quantitatively the previous plot. Replacement of tensorial elastic stress field by scalar function is based on questionable assumption (see further) that viscous dissipation is of the order of elastic energy

Short review on turbulent drag reduction

Main conjectures and predictions of theories of turbulent drag reduction

1. A polymer in turbulent flow is influenced only by small scales, below dissipation one, where flow is random in time and spatially smooth- ***similar to elastic turbulence***
2. Polymer coil-stretch transition can occur in random flow, if $Wi = \lambda / \lambda_{turb} > 1$ in full ***analogy with elastic turbulence***
3. Stretched polymers create elastic stresses, which can balance Reynolds stresses in a bulk turbulent flow at some scale and above some polymer concentration
4. First quantitative theory, which take into account the elastic stresses, made predictions about elastic waves and spectra and ***links turbulent drag reduction problem to elastic turbulence and mhd*** (Lebedev et al 2000, 2001, 2003) for linear model of polymers and in unbounded flow

Numerical simulations of the last decade give a great insight into the problem:

1. Observation of drag reduction in full scale numerical simulations in a turbulent channel flow of Oldroyd-B model of viscoelstic fluid (Beris et al 1997 and later on)
2. Role of elastic stresses and coil-stretch transition in drag reduction in turbulent Kolmogorov flow, which reflects in universal function for the friction coefficient (Boffetta, Celani, Mazzino 2004): $f(\mathrm{Re}^{\alpha}, Wi^{\beta})$
3. Stretching of polymer in turbulent shear f low (Davoudi, Schumacher 2006) and recently (T. Watanabe, T. Gotoh 2010; F. Bagheri et al 2010)

Numerical simulations of the last decade give a great insight into the problem:

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(ii) role of elastic stresses and coil-stretch transition in drag reduction in turbulent Kolmogorov flow, which reflects in universal function for the friction coefficient $f = F(\mathrm{Re}^{1/3}, Wi^{2/3})$ (Boffetta, Celani, Mazzino 2004)

(iii) Stretching of polymer in turbulent shear f low (Davoudi,Schumacher 2006)

Theory:

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What experimental data and theoretical predictions are lacking to make further progress in the field?

- Measurements and characterization of polymer stretching and elastic stresses and a possibility of the coil-stretch transition in random and turbulent flows
- Scaling of elastic stresses with Re and Wi. Development of a new stress sensor to measure directly statistics of elastic stresses locally and their spatial distribution in elastic turbulence and further in turbulent flow
- Theory that makes quantitative predictions about scaling relations of the friction factor with Re and Wi: $f(\mathrm{Re}^{\alpha}, Wi^{\beta})$
- Dependence of the turbulent drag reduction onset and value on polymer molecular weight, elastic nonlinearity and concentration
- Distribution of elastic stresses in random and turbulent flows

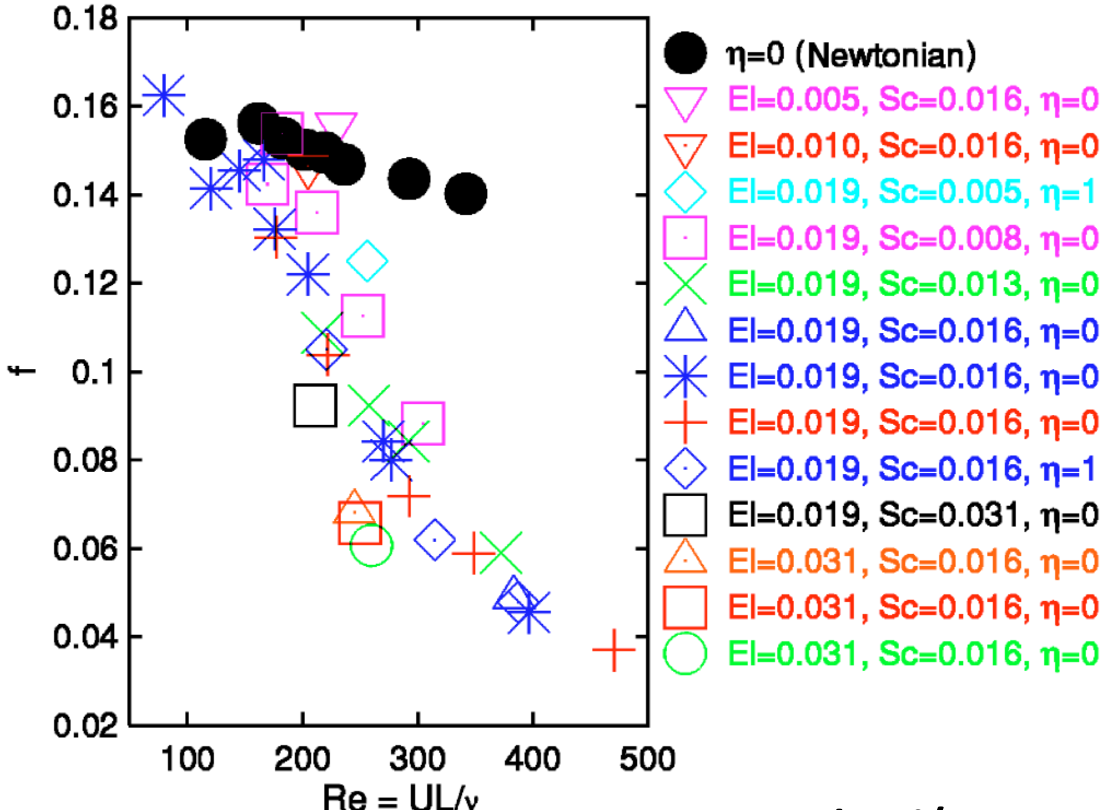
- ***Conclusion***: measurements and characterization of elastic stresses in turbulent flow are the key issues to solve the turbulent drag reduction problem (two fields should be measured like V and H in MHD)

Drag reduction in driven turbulent Kolmogorov flow

G. Boffetta, A. Celani. A. Mazzino
Phys. Rev. E 71, 036307 (2005)

Drag coefficient f=FL/U^2

At Re=UL/ν>50 the flow is turbulent



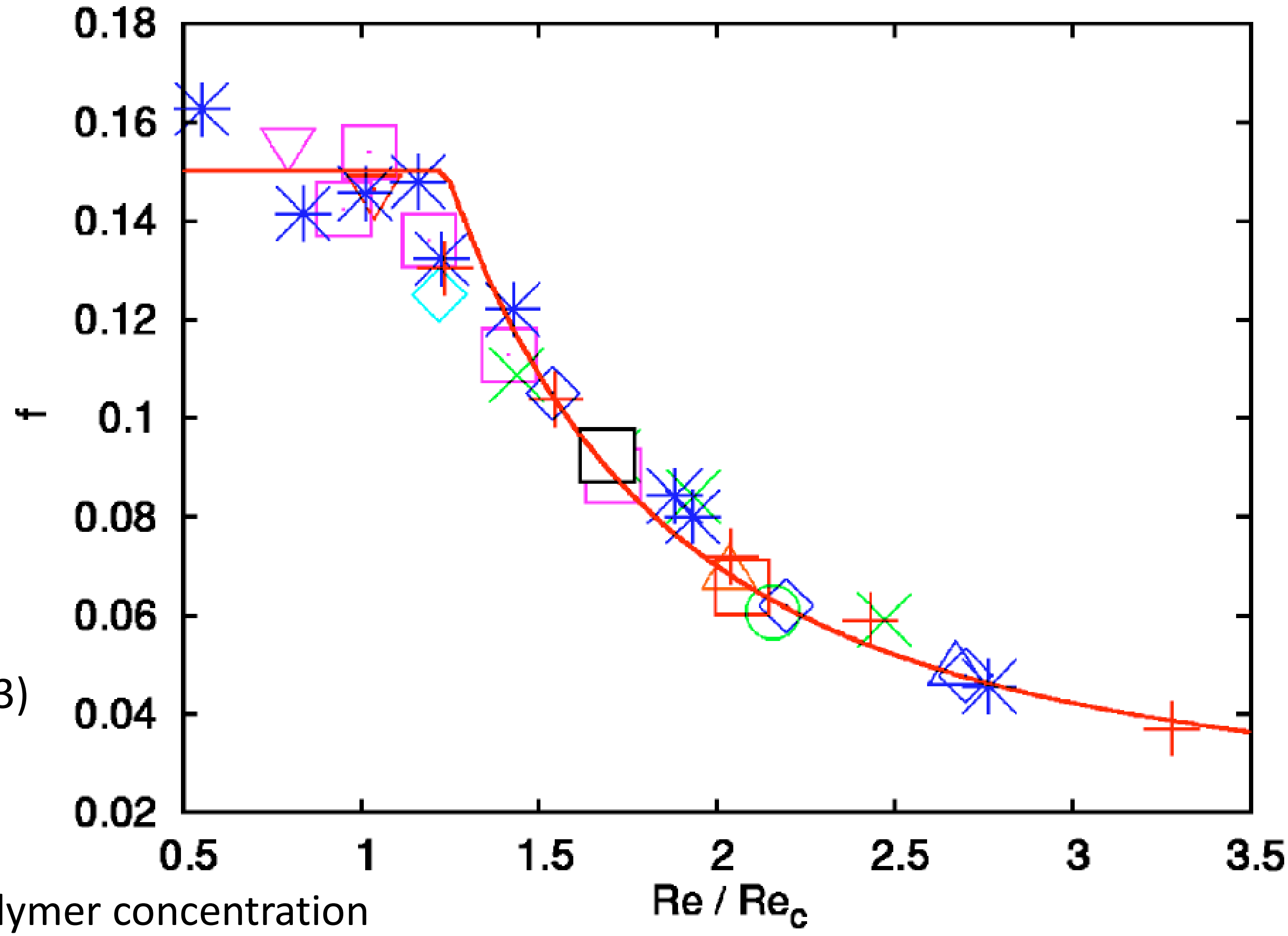
El=Wi/Re

Numerical simulations without boundaries- **bulk effect!**

$Re_c \propto [\![(2/El+1/Sc)]\!]$^(2/3)

At Sc→ ∞ $Re_c \propto El^{-2/3}$

Also in theory by
Fouxon &Lebedev (2003)

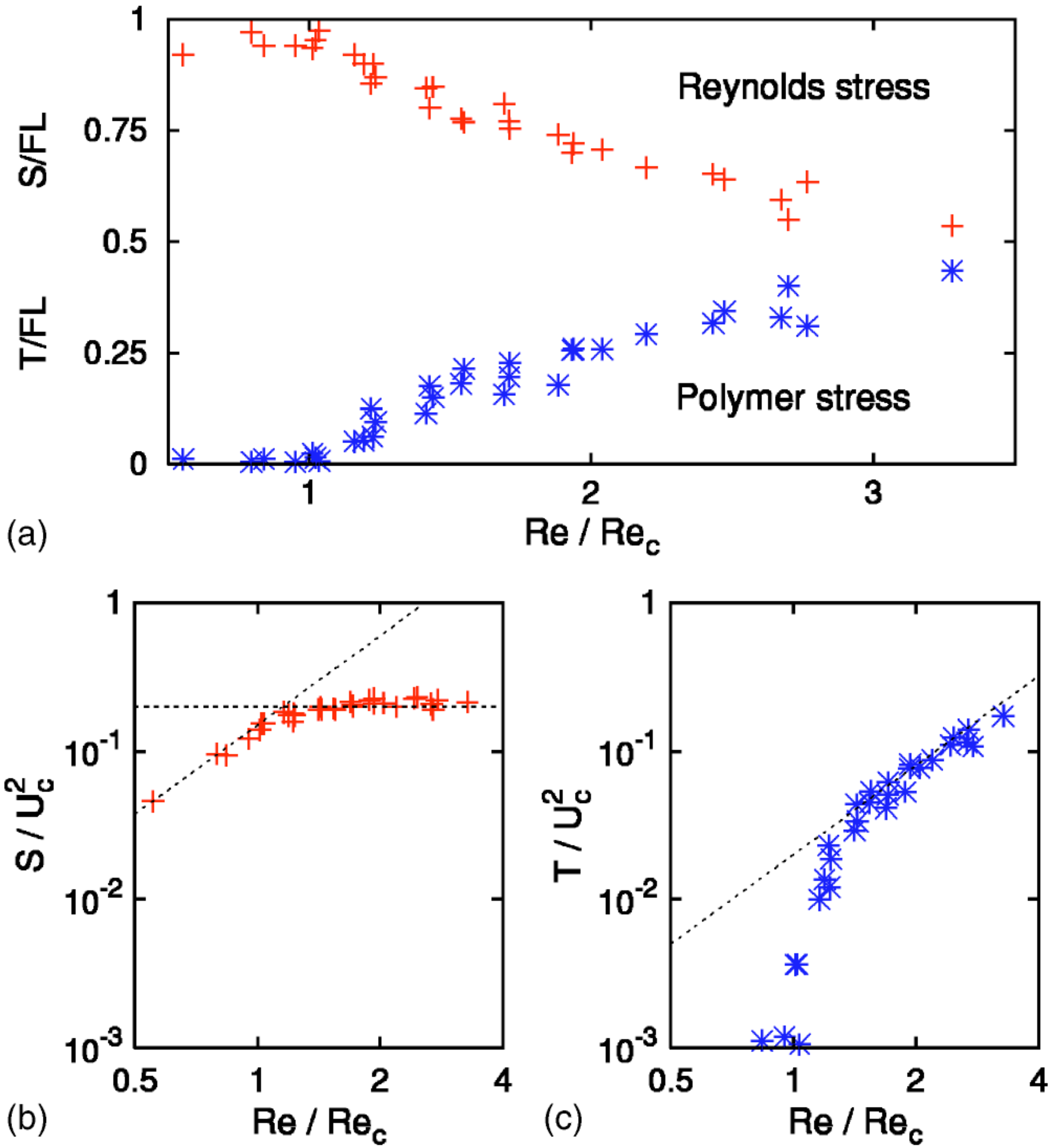


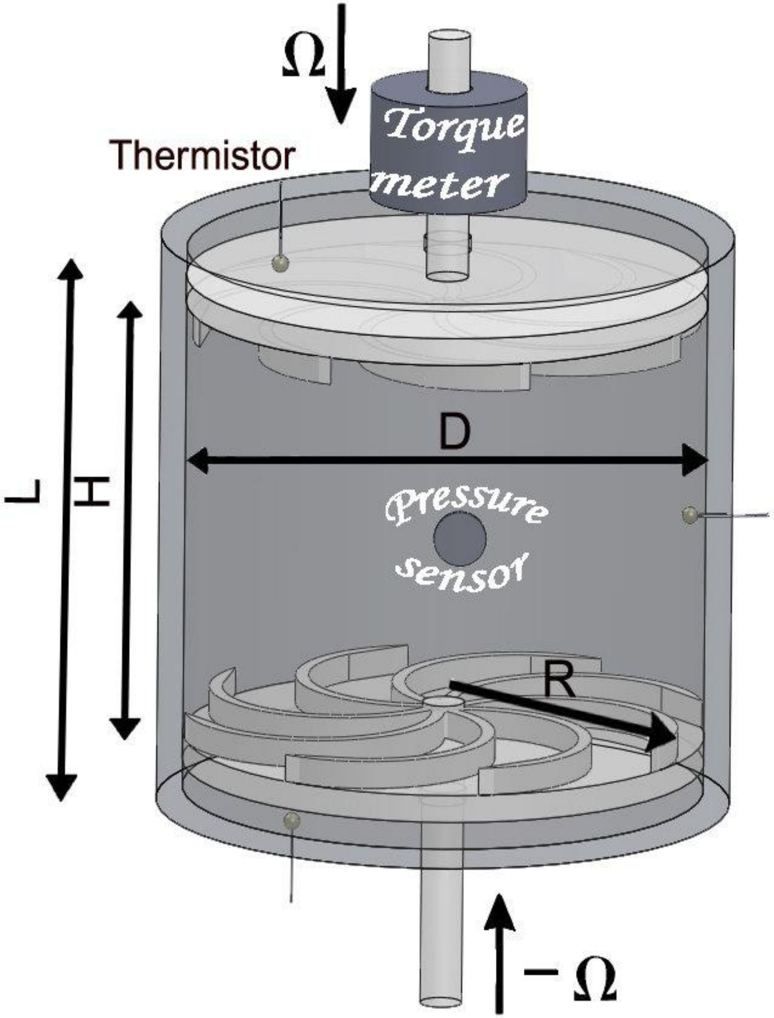
Dependence of Re_c on polymer concentration suggested to be as $C^{-2/3}$for linear polymer

Drag coefficient as a function of the rescaled Re

The solid line is the fit f= $\begin{cases} \beta & \text{Re<Rec} \\ \gamma \left(\frac{Re_c}{Re}\right)^2 + \delta & \text{Re>Rec} \end{cases}$

Relation between stresses





Experimental setup

D=0.29m, L=0.310m
R=0.132m, H=0.210m
8 blades 10mm height

2 brushless motors 13Nm each, 0-540 rpm
Torque meter noncontact calibrated
MCRT 49001 up to 11.3 Nm 0.1% acc

Calibrated **pressure sensor** PCB106B50
Up to 35kPa with 4.8x10^-4 kPa resolution

Polymer PAAm 18Mda of 5,10,20,40,80 ppm
And 20 ppm in water-sugar solvents with 14,20, 24,28,31,34,36% or η=1.534, 1.945, 2.331, 2.855, 3.475, 4.052, 4.621 mPas
Mixing is carried out in the system ~1h at 15 rpm

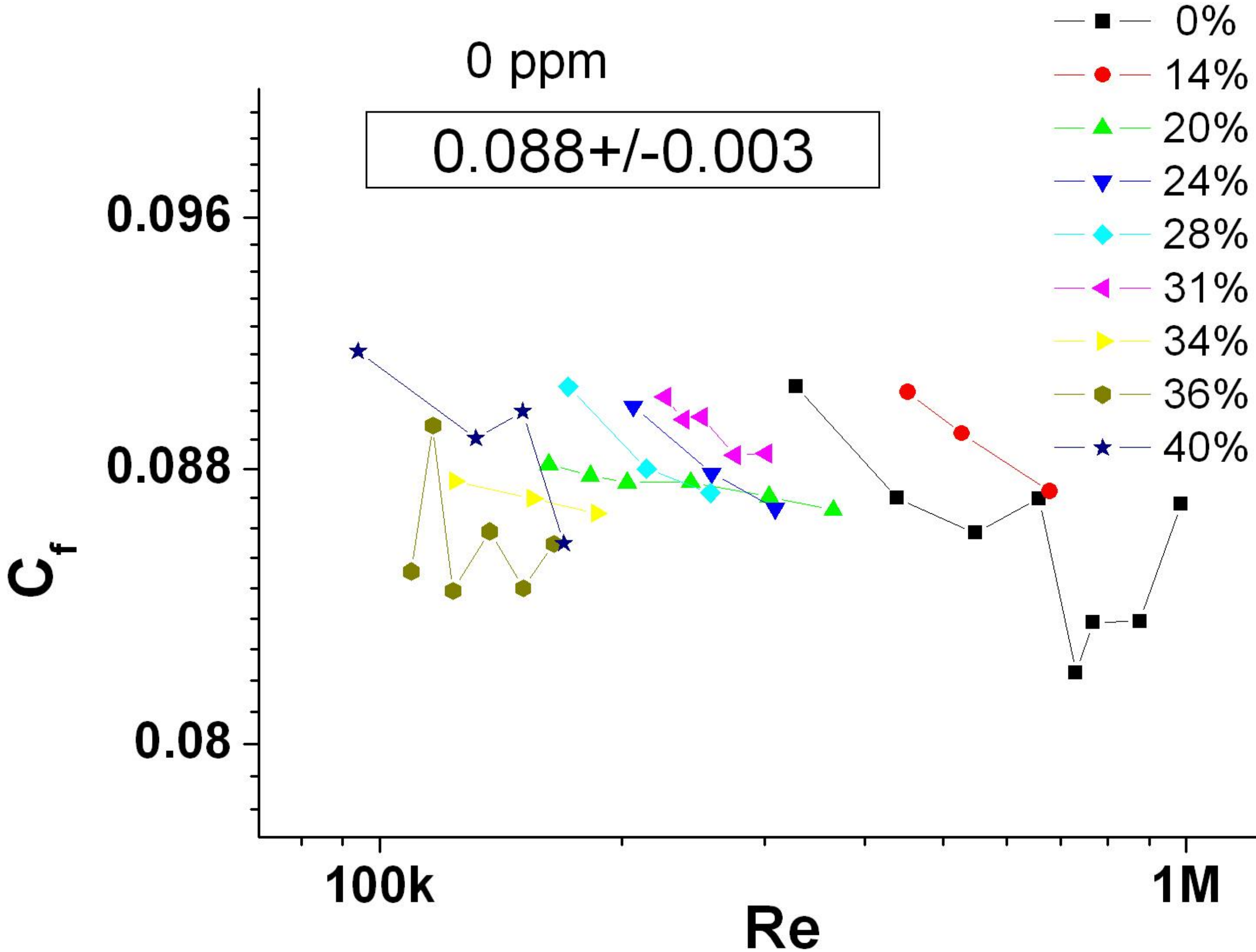
We varied polymer concentration ***C*** and ***El=Wi/Re=***$\frac{\lambda\eta}{\rho R}$ (also $\lambda \sim \eta$)

Characterization of turbulent flow without polymers

Turbulent regime occurs at Re in the range [10^5-10^6]

In this range we can defined the main characteristic of turbulent flow from two measured quantities: Γ and p

1. $R_\Lambda = p_{rms}\sqrt{V/\eta\Gamma\Omega}$ and changes in the range [70-250]
2. $\Lambda/R = R^{-1} p^{1/2}{}_{rms}\sqrt{V\eta/\rho\Gamma\Omega}$ and changes in the range [0.0018-0.0064]
3. $\xi/R = R^{-1}\left(\frac{V\eta^3}{\rho^2\Gamma\Omega}\right)^{1/4}$ and changes in the range [$8\mathrm{x}10^{-5}$-$4.5\mathrm{x}10^{-4}$]
4. $L^*/R = \frac{V}{R\Gamma\Omega}(\rho p_{rms})^{3/2}$ and changes in the range [0.65-0.95]



Friction coefficient C_f for pure solvents versus Re in **turbulent region** Re>100k.

C_f is about constant for all solvents.

Goal of the experiment and main technical achievement

- The main goal of the experiment is to vary elastic stresses and the corresponding parameter ***Wi*** by changing solvent viscosity and in this way polymer elasticity

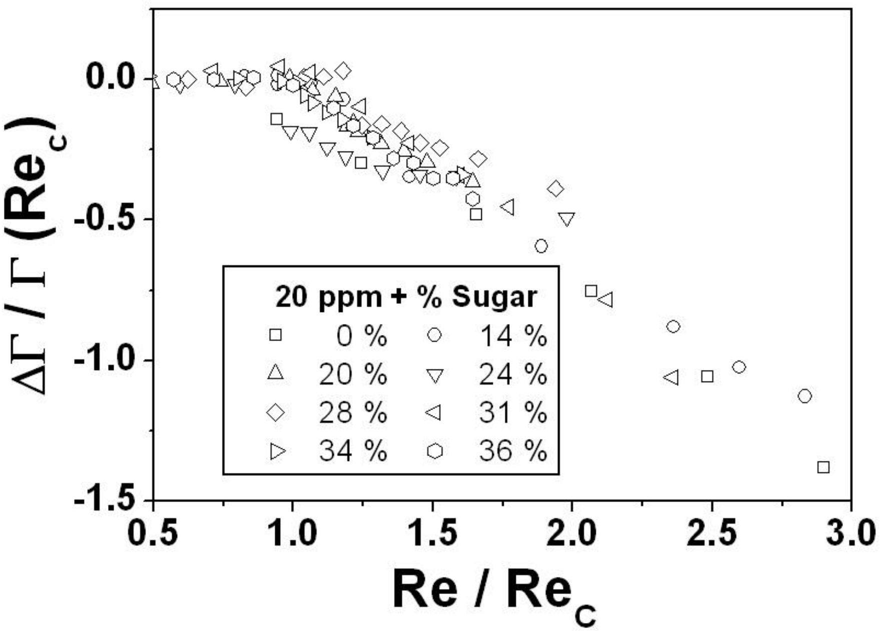
- We are still not able to measure elastic stresses but we investigate the influence of ***Wi*** and polymer concentration ***C*** on the onset of DR

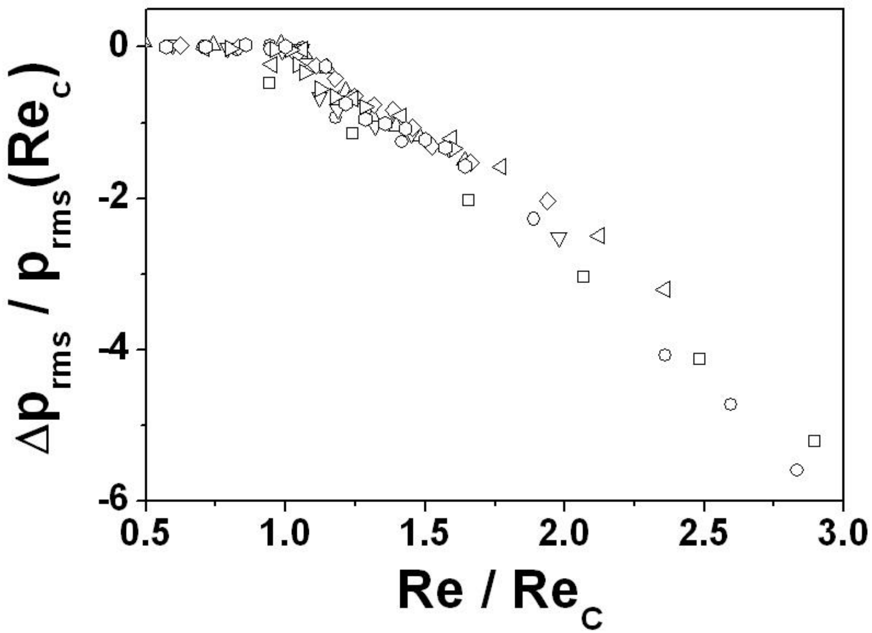
- Another question is: what is the nature of this transition?
  And whether the behavior of the friction coefficient is universal.

- And finally, what is the influence of ***Wi*** and ***C*** on power spectra and statistics of measured torque and pressure?

- The main technical achievement in the experiment is long term stability of polymers in turbulent flow that allows to take large data sets up to 10^6 data points and up to 3.5 hours without polymer degradation.

Reduced average torque vs Re　　Reduced pressure fluctuations vs Re



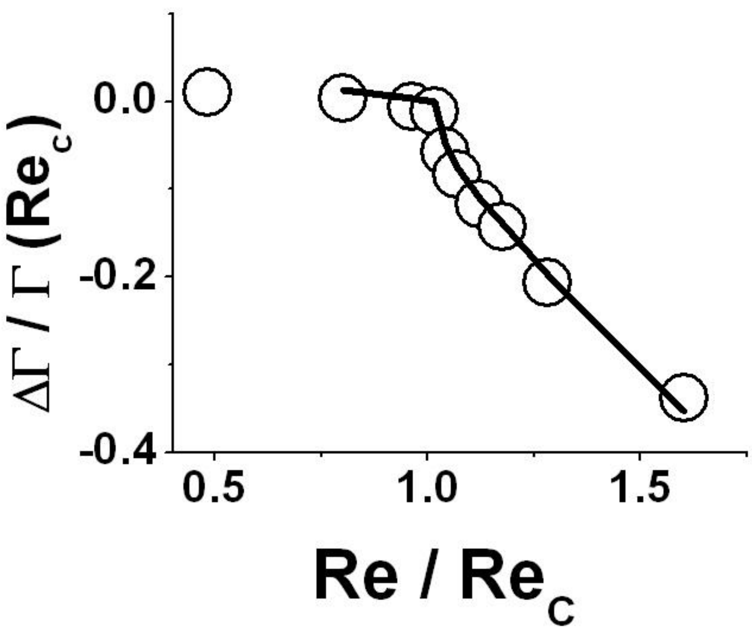


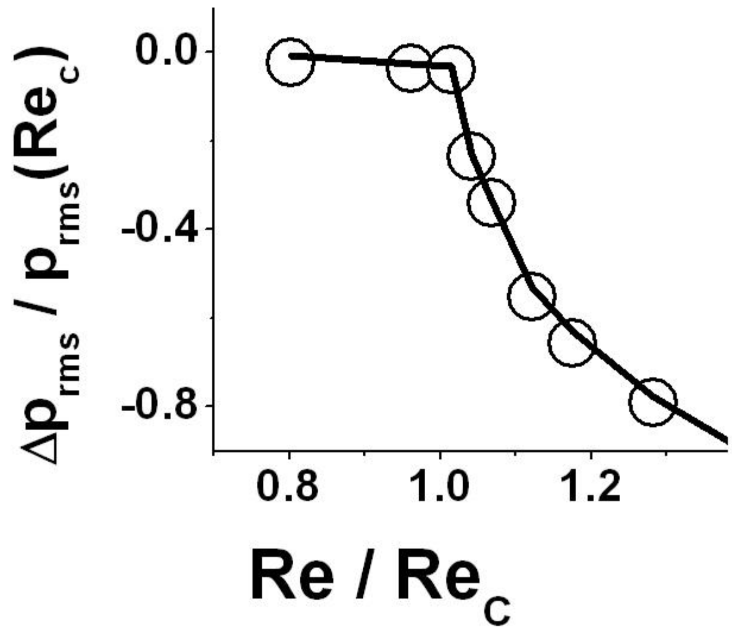
ΔΓ=Γ_{20}-Γ_0, Re_c(η)

Reduced average torque and pressure fluctuations for 8 values of El (solvent viscosities) vs Re/Re_c

Reduced average torque vs Re

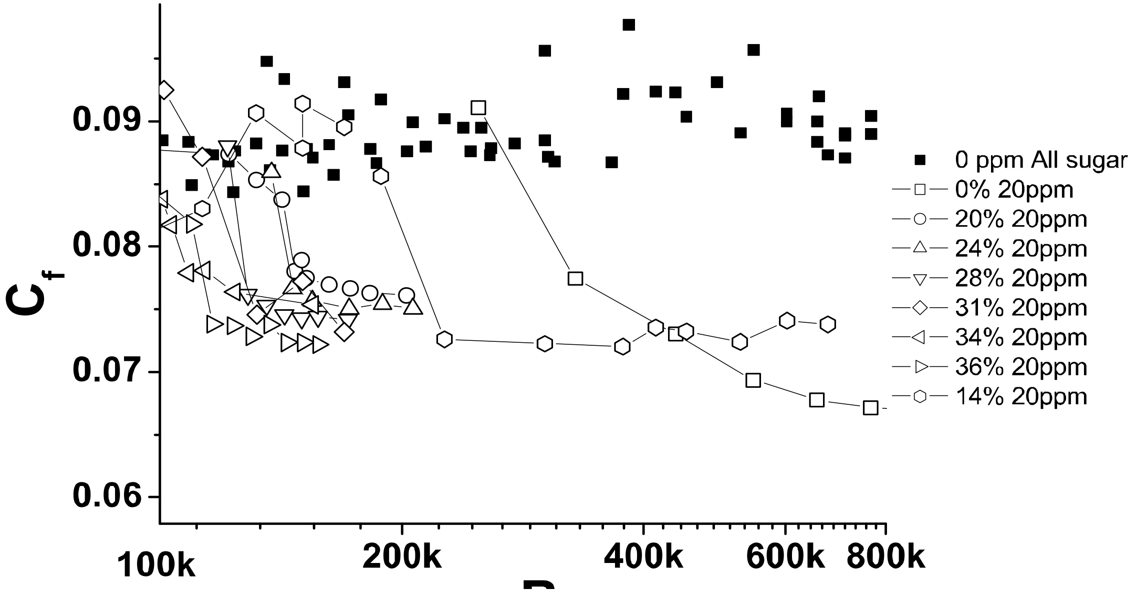
Reduced pressure fluctuations vs Re



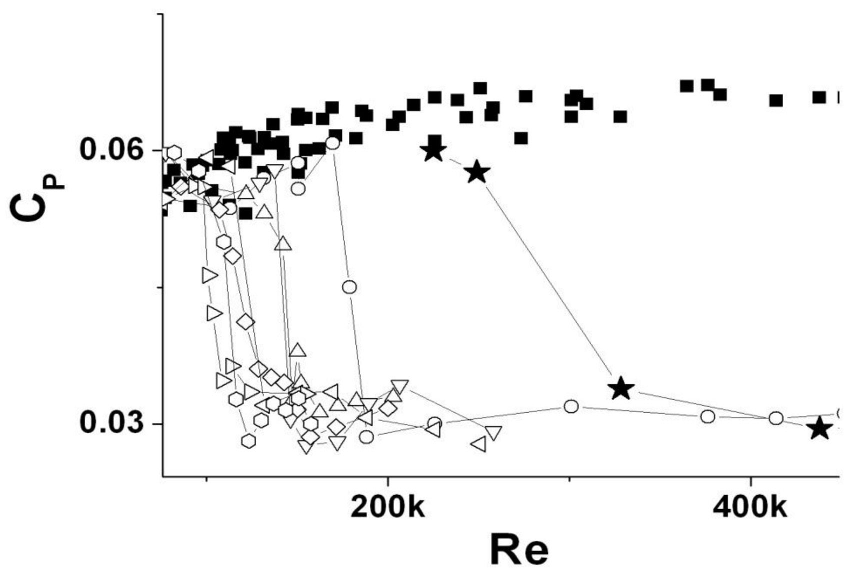


20 ppm PAAm and 36% sugar water solvent.
Solid curves are the fit to the data by y∼√x.
It is a *forward bifurcation* to the *Drag Reduction state*

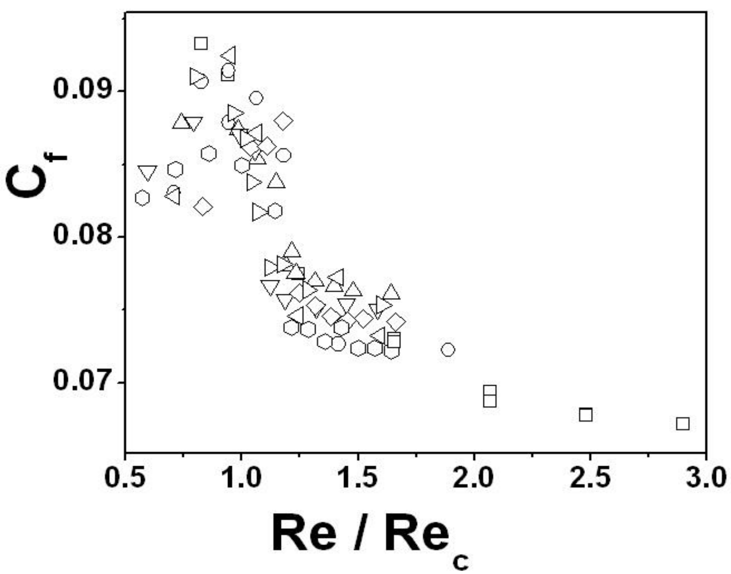
Friction coefficients as a function of Re for different EI

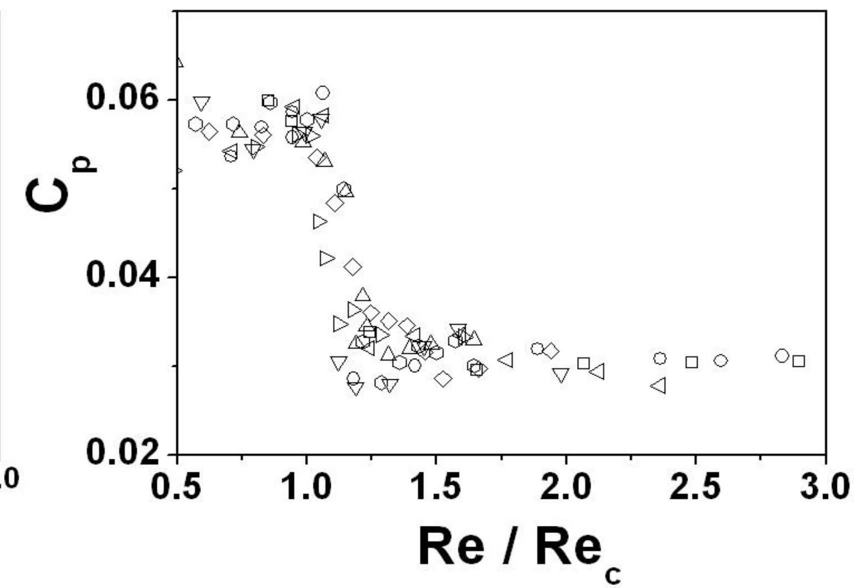


$C_f = \Gamma / \rho R^5 \Omega^2$



$C_p = p_{rms} / \rho R^2 \Omega^2$



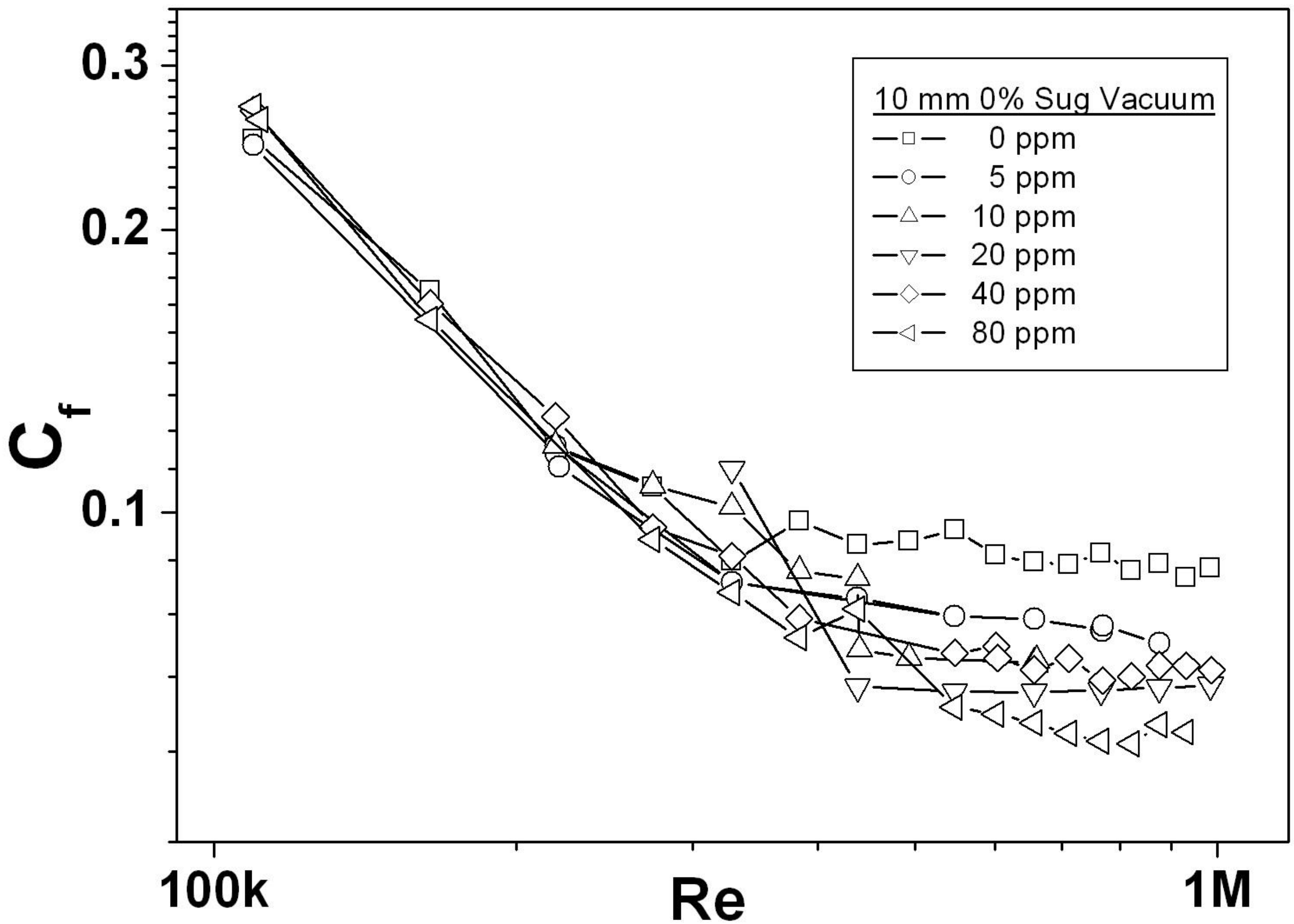


Friction coefficients as a function of rescaled ***Re*** for different ***El***

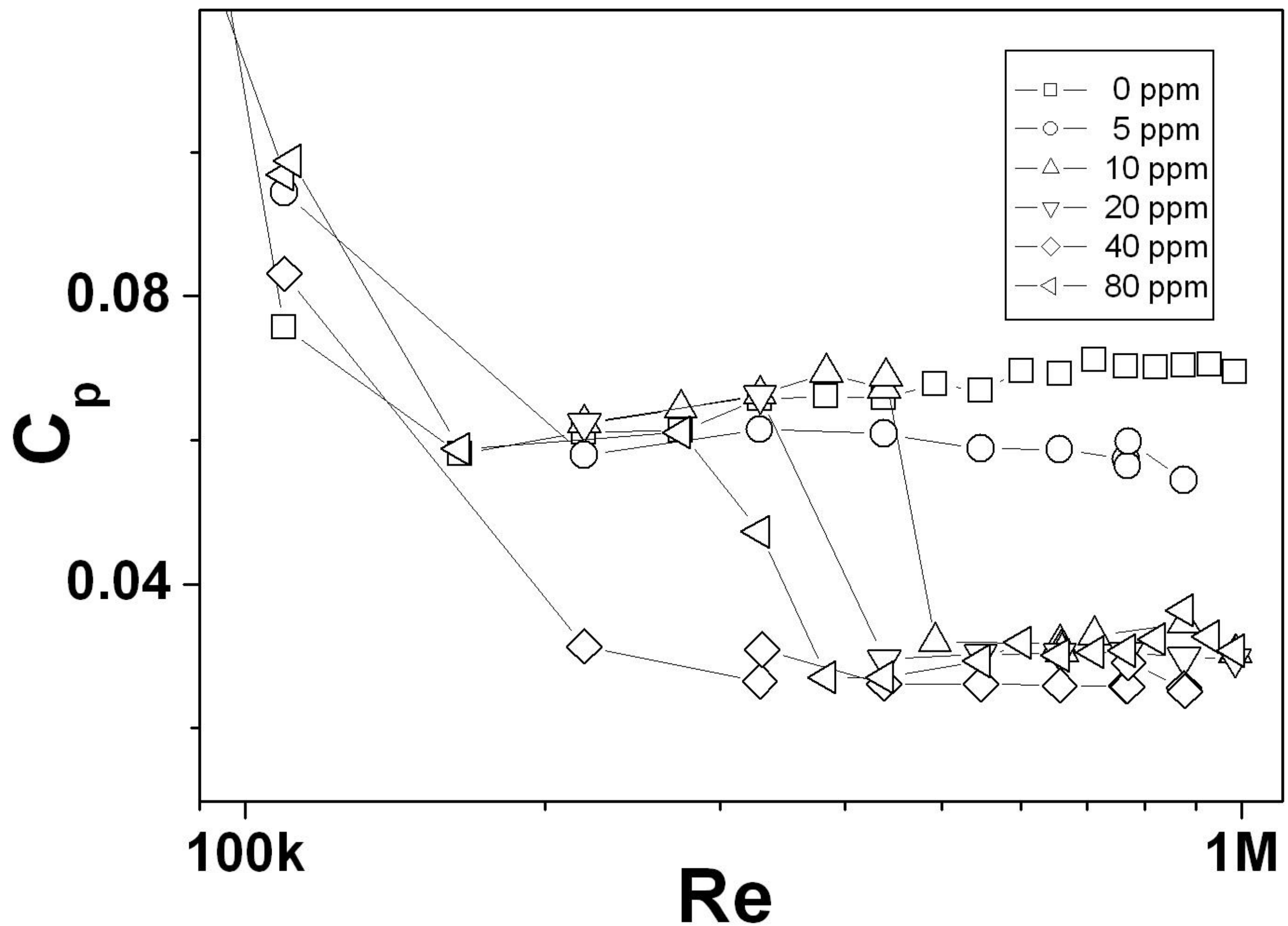
Universal dependence for all values of ***El***

Max DR is about 27% for $\boldsymbol{C_f}$ and about 50% for $\boldsymbol{C_p}$

11/17/2011

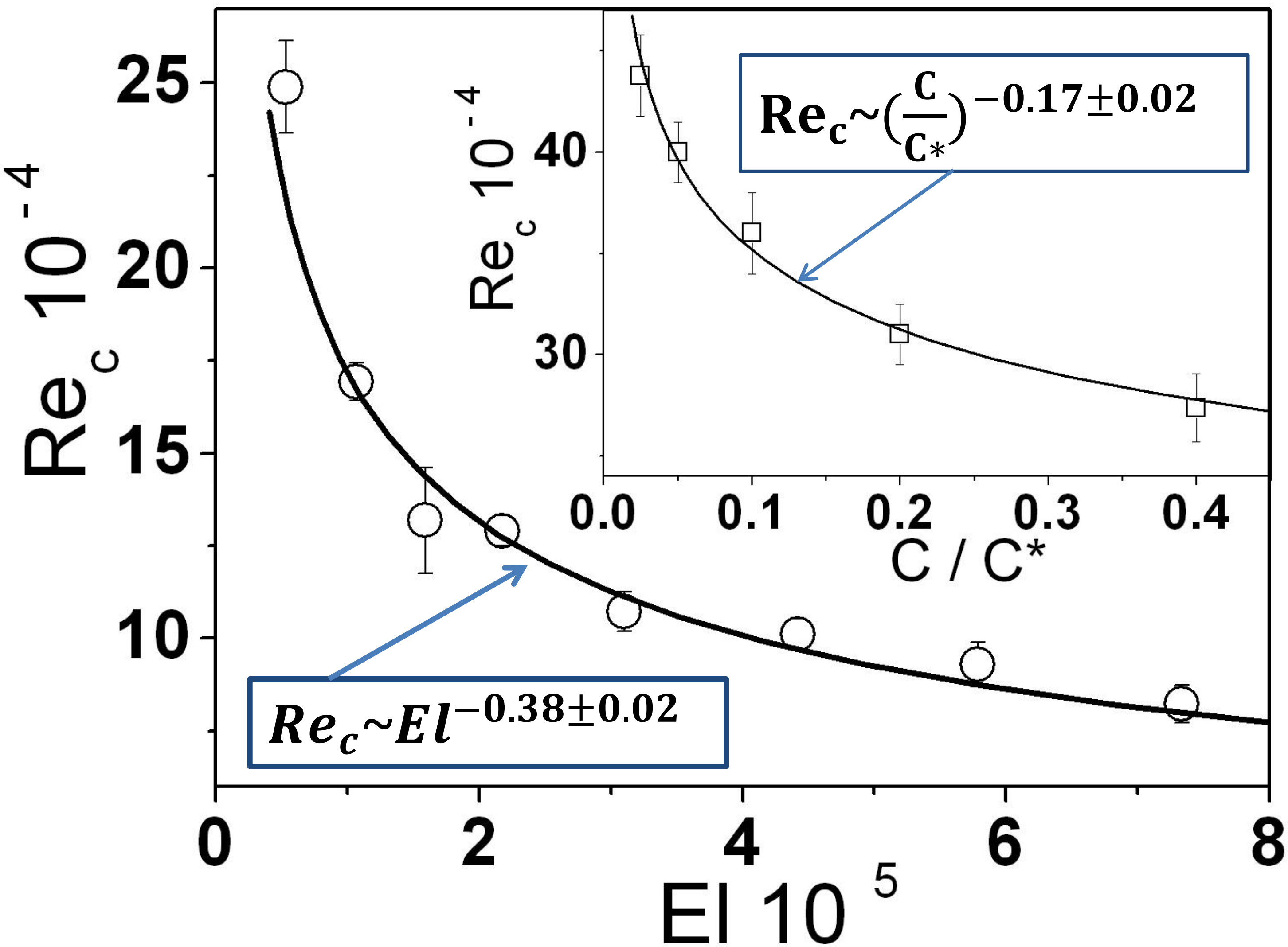


Friction coefficient C_f as a function of Re
for different polymer concentrations C
Max DR is about 37% for 80 ppm solution



Friction coefficient C_p as a function of Re
at different polymer concentrations

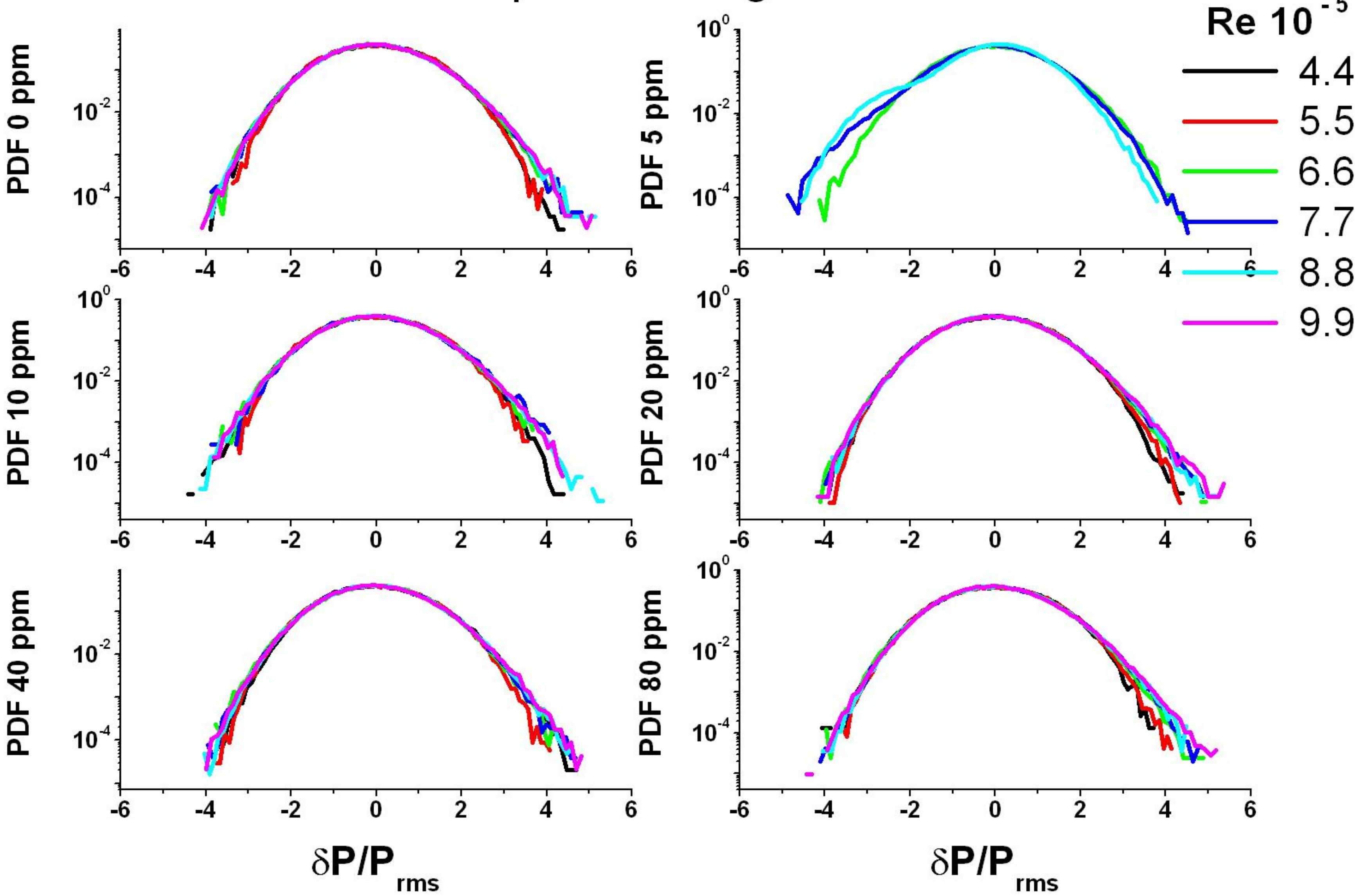
Max DR is about 67% for 80 ppm polymer solution

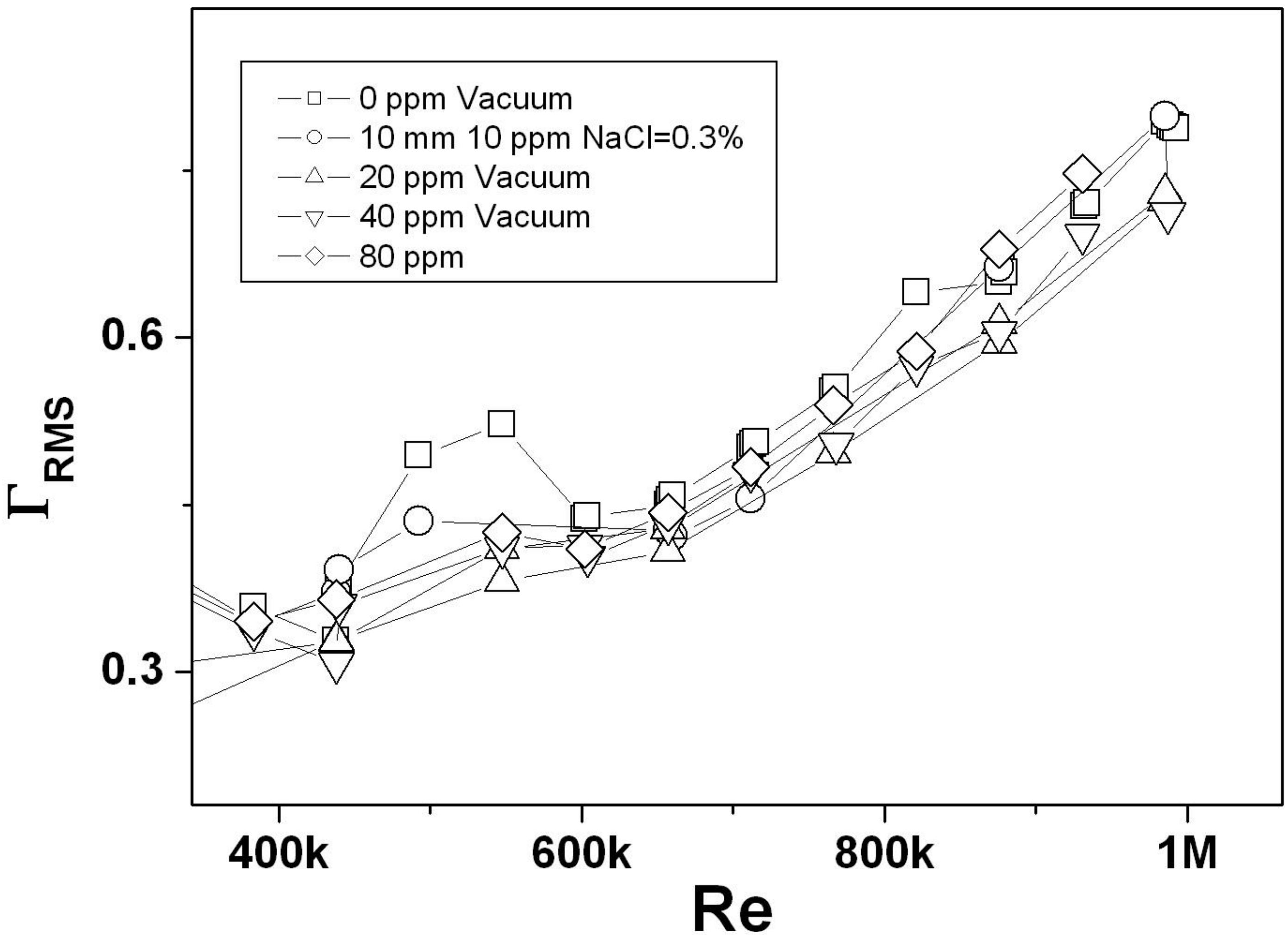


Dependence of Re_c at the onset of turbulent drag reduction on El and reduced polymer concentration ***C/C****.

C*=200 ppm is overlap concentration for 18Mda PAAm

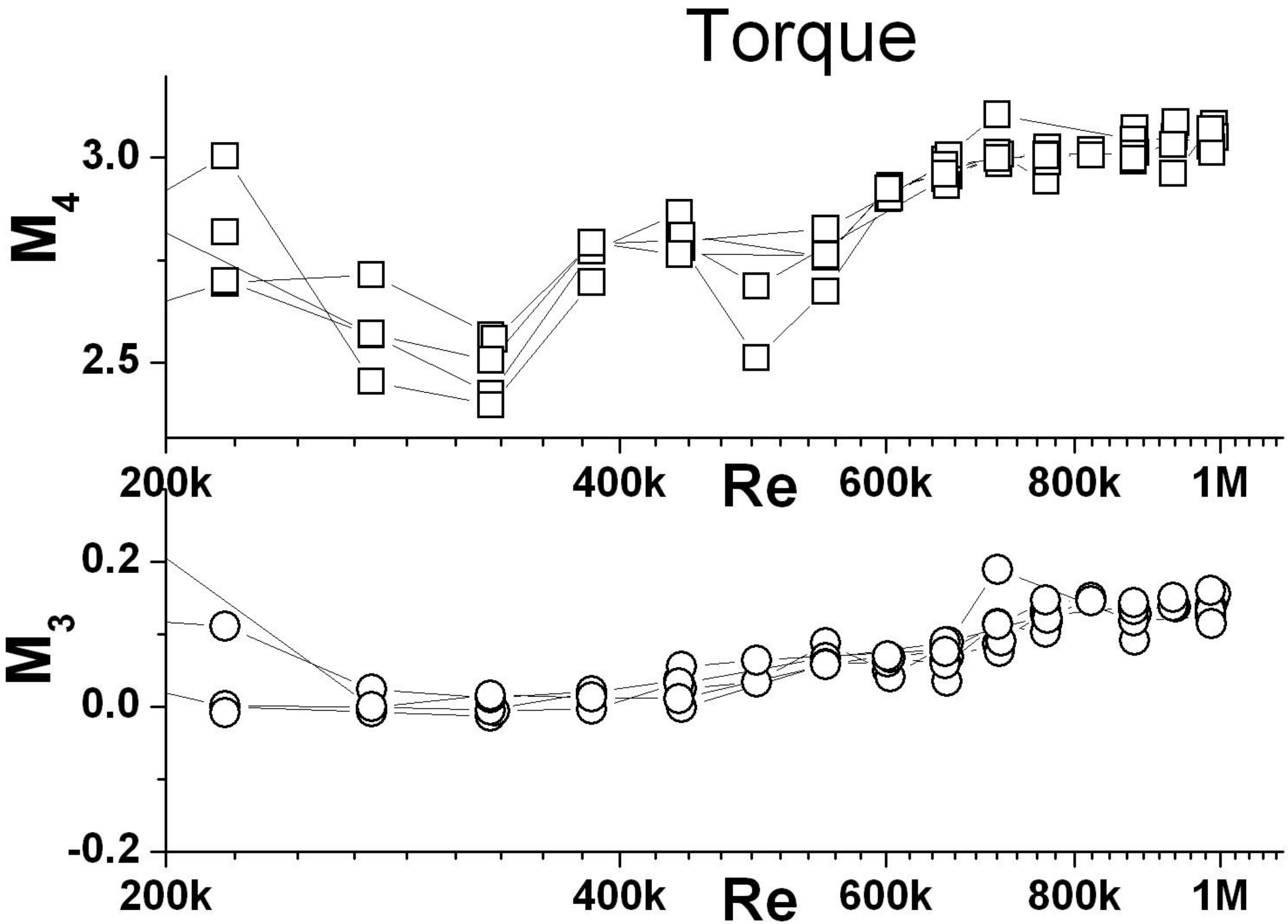
Torque 0% Sugar statistics





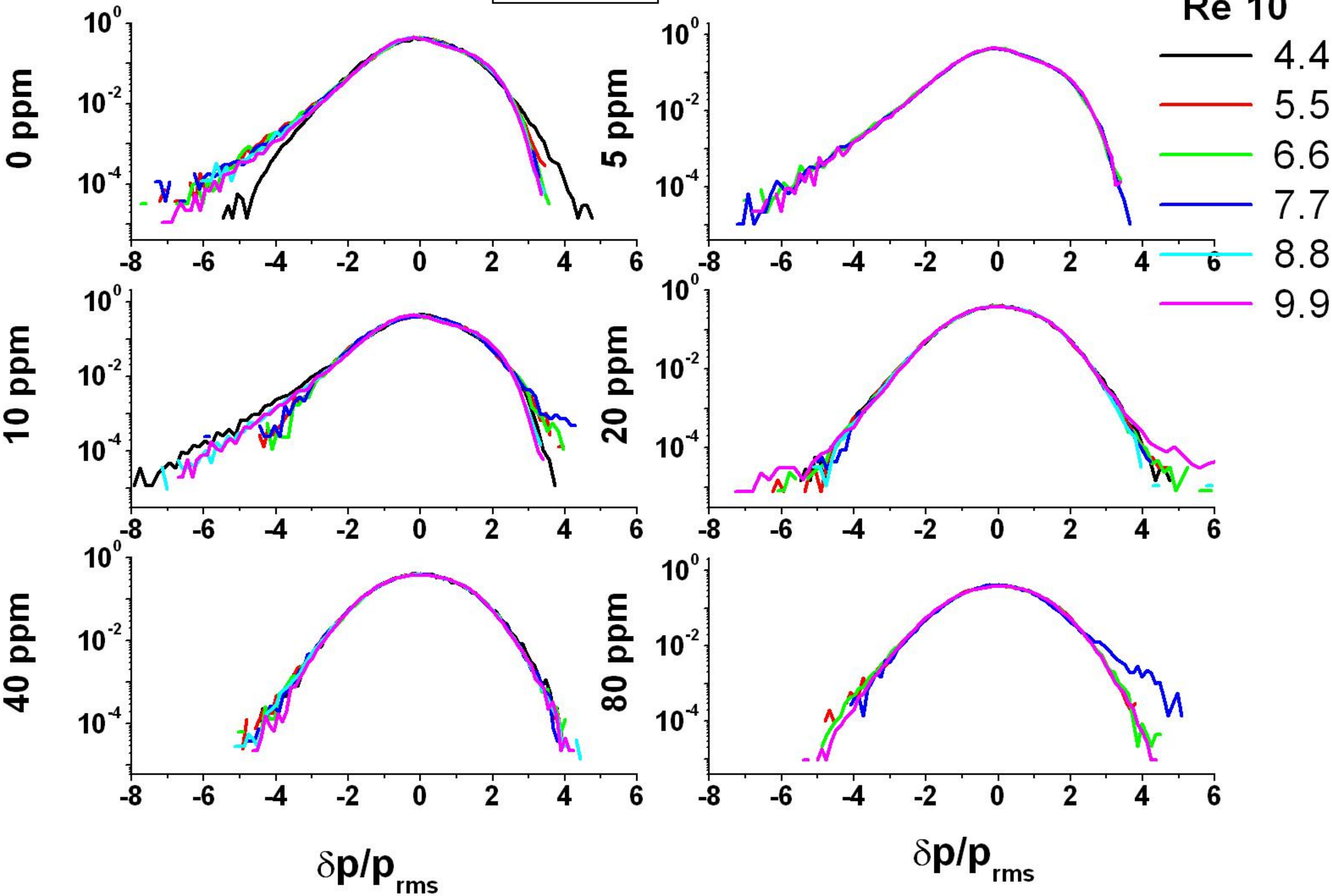
Γ_{rms} grows with ***Re*** for all polymer concentrations

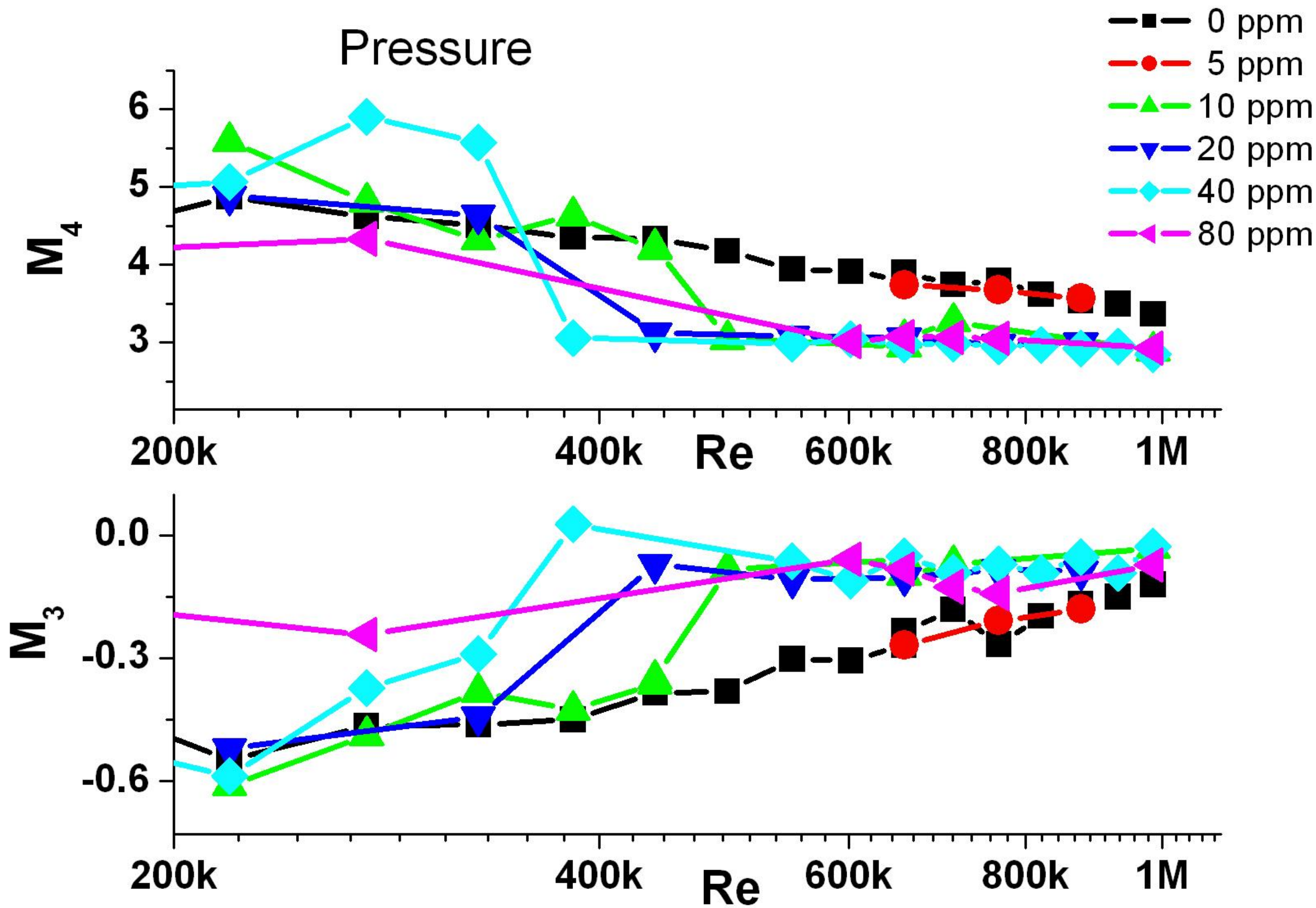
Similar dependence is observed at different ***EI***



Similar variations in M_3 towards 3 and M_4 towards 0.2 are observed due to Re increase for different El

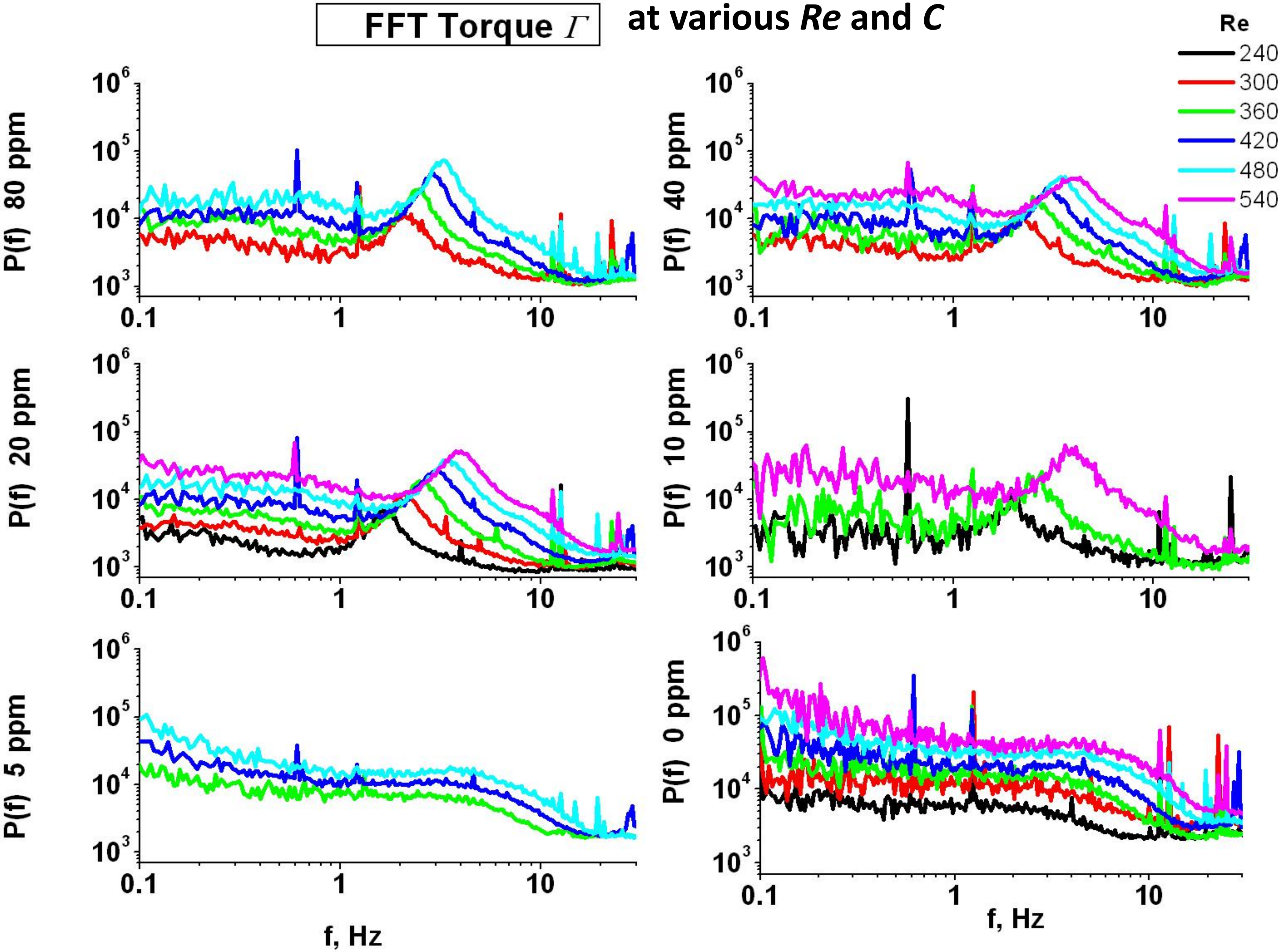
Pressure statistics



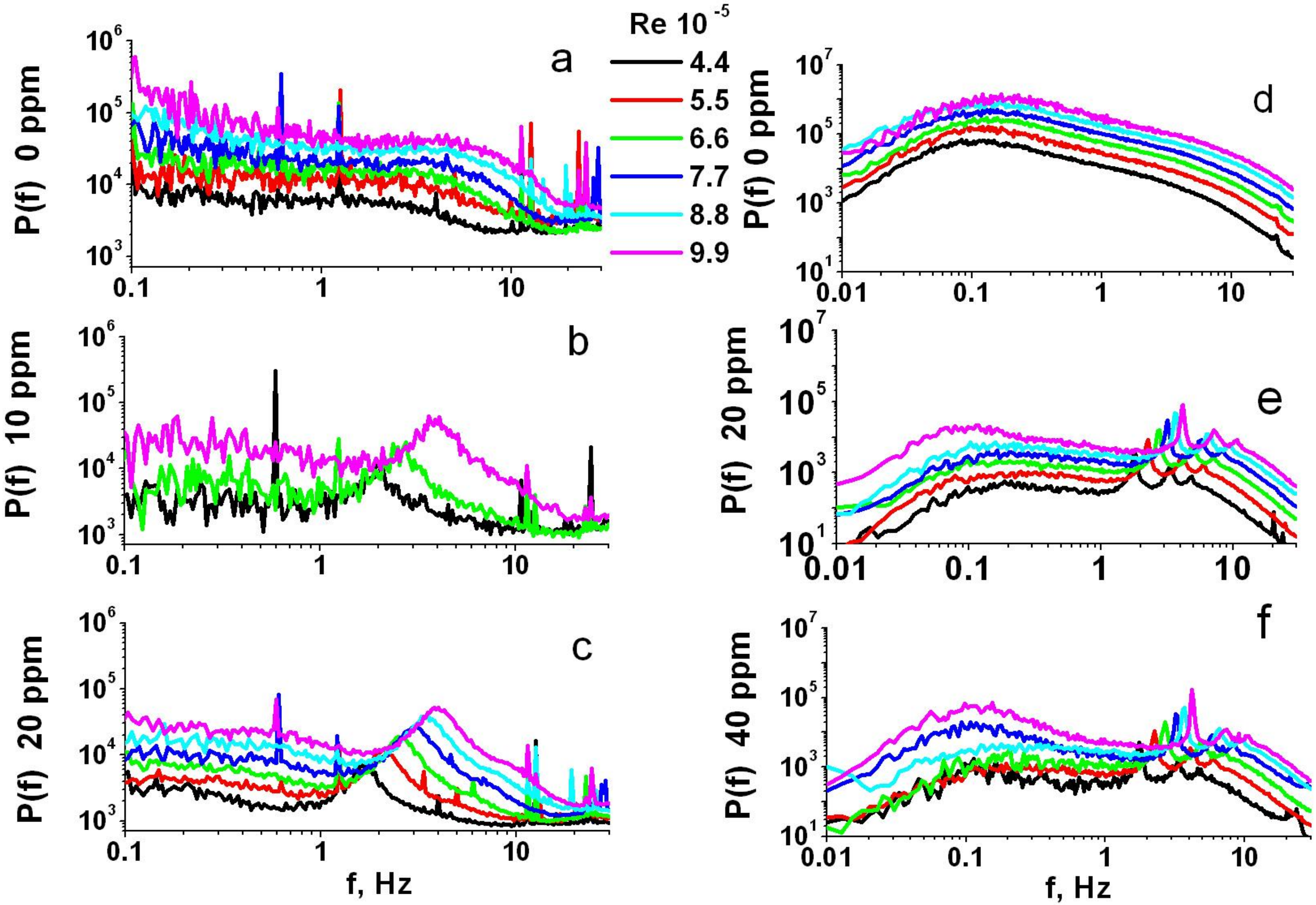


Pressure
0 ppm
5 ppm
10 ppm
20 ppm
40 ppm
80 ppm
M_4
6
5
4
3
200k
400k
Re
600k
800k
1M
M_3
0.0
-0.3
-0.6
200k
400k
Re
600k
800k
1M

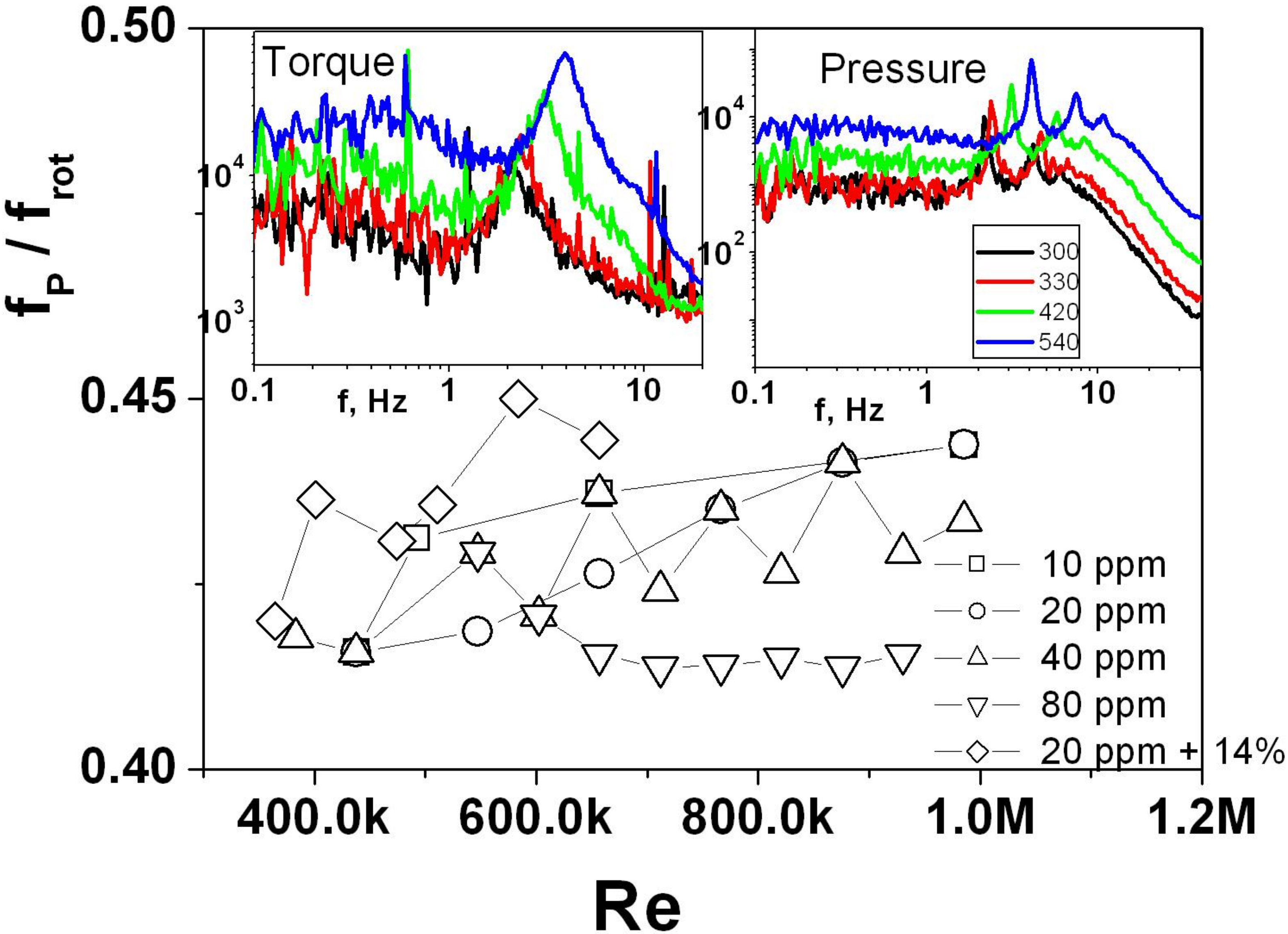
FFT Torque Γ at various *Re* and *C*

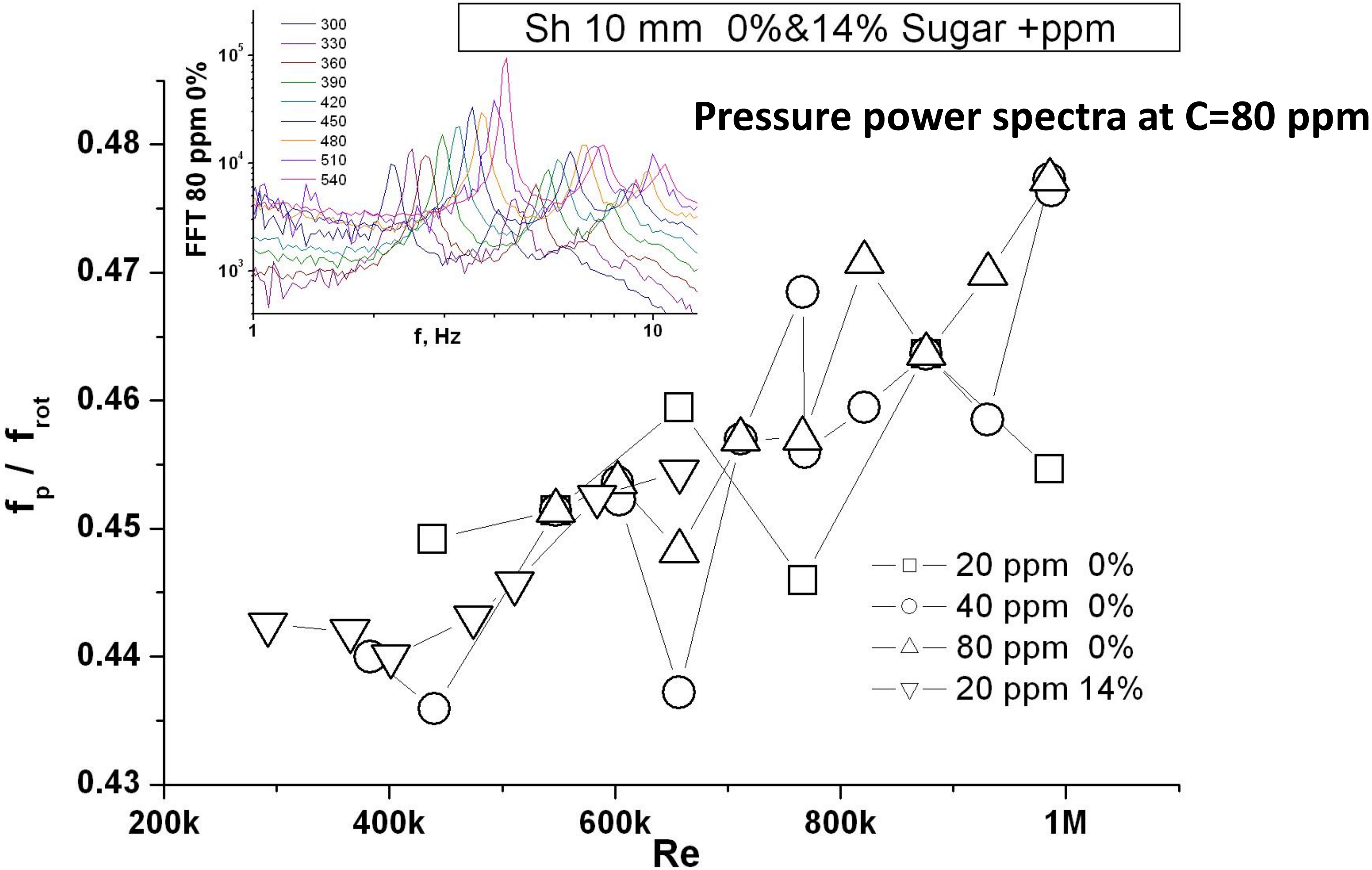


Frequency power spectra of torque and pressure at various *Re* and *C*



Frequency power spectra for torque and pressure at *C*=20 ppm and 14% sugar





Frequencies of peaks in the pressure power spectra
at different polymer concentrations ***C***

Summary of the results

- Significant DR in swirling flow produced by disks with 8 blades is observed- it is bulk effect, though comparable DR in flow with smooth disks is also found-boundary effect (?) (50% at ***C***=15 ppm compared with about 30% at ***C***=20 ppm)

- Both friction coefficients $\boldsymbol{C_f}$ and $\boldsymbol{C_p}$ show universal dependence versus ***Re***$\boldsymbol{/Re_c}$ for different El though in odds with the predictions

- Transition curves to the DR state are described by the power laws ***Re***$\boldsymbol{\sim El^{-0.38}}$ at ***C***=20 ppm and ***Re***$\boldsymbol{\sim(C/(C*)^{-0.17}}$ ***El***=const and both differ from the predictions

- Frequency power spectra of both $\boldsymbol{\Gamma}$ and $\boldsymbol{p}$ reveal dramatic interplay between orders of magnitude reduction of low frequency modes and the emergence of a well-defined peak at $\boldsymbol{f_p}$ with increasing ***C*** and ***El.*** These peaks are related to the main vortex rotation frequency
- The transition to DR state occurs for significantly stretched polymers (further)

Appropriate *Wi* to characterize polymer stretching

1. Global $Wi=\lambda\Omega$ is based on integral velocity $v=R\Omega$ and spatial scale R. So it is not directly related to the characterization coil-stretch transition (CST). Its whole range of changes is [1-10] and near Re_c[2-5].
2. Wi based on the wall stresses $\tau_w=3\Gamma/2\pi R^3$ (for swirling flow) is written as $Wi_\tau=\lambda\tau_w/\eta$. Its whole range of changes is [1-200] and near Re_c [10-100].
3. Wi based on Kolmogorov scales (space and velocity) is $Wi_\xi=\lambda/t_\xi$ and $t_\xi=\xi/u_\xi=(\eta/\rho\varepsilon)^{1/2}$, where the energy dissipation rate is $\varepsilon=\Gamma\Omega/\rho V$, $V=\pi R^2 H/2$. Thus $Wi_\xi=\lambda(\Gamma\Omega/\eta V)^{1/2}$ and its whole range of changes is [1-300] and near Re_c [100-250].
4. Local Wi can be defined as $Wi_{loc}=\lambda\rho u_{rms}^2/\eta$ and $u_{rms}^2=p_{rms}/\rho$. Thus $Wi_{loc}=\lambda p_{rms}/\eta$ with its whole range of changes [1-100] and near Re_c [15-50].

Thus, all defined *Wi* >>1 that means polymers are strongly stretched nearRe_c

Elastic stresses are the source of flow modifications

Suppression of low frequency modes that leads to the emergence of the peak is the result of elastic stresses We will support this claim by N-S eq with addition of polymer stress tensor σ_{ij}. One obtains from N-S

$$\frac{\Delta p}{\rho} = -\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}(V_i V_j - \sigma_{ij})$$

At fixed Re and increasing ***Wi*** one can gets $\boldsymbol{V_i V_j \approx \sigma_{ij}}$ that leads to decrease of pressure fluctuations what is observed in the ***p***-power spectra. Similar decrease of p-fluctuations occurs due to increase of ***C***: the larger ***C***, the larger $\boldsymbol{\sigma_{ij}}$ for the same polymer stretching (***Wi***) since $\mathbf{\sigma_{ij} \sim f(C)}$

Conclusions

- The transition lines to DR state are obtained in $\boldsymbol{Re_c}$-***El*** and $\boldsymbol{Re_c}$-**C/C*** parameter space
- The transition is continuous, forward bifurcation with universal scaling above the transition
- The friction coefficients show universal behavior above the onset of DR for all ***Re*, *El*** and ***C***
- PDFs of $\boldsymbol{\Gamma}$ and $\boldsymbol{p}$ are collapsed in scaled variables for all ***Re*** for each ***C*** and ***El***
- Power spectra for both $\boldsymbol{\Gamma}$ and p are strongly altered by changing ***C*** and ***El***
- We suggested arguments to explain that these effects are result of increasing elastic stresses due to increasing ***El*** and ***C*** at constant ***Re***